Nothing But Coincidences

Adventures and Misadventure of Einstein's Point-Coincidence Argument

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UNIVERSITÀ
DEGLI STUDI
DI TORINO

Introduction

1916. №. 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. ***Die Grundlage der allgemeinen Relativitätstheorie;***
von A. Einstein.

Introduction

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point-coincidence argument (John Stachel)

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- the relationships between spacetime coincidences are independent of the coordinate system

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general covariance = no privileged coordinate systems

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point-coincidence argument answer to the hole argument

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- The hole argument extremely successful
- The point-coincidence argument fundamentally neglected

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nothing but coincindeces = central message of general relativity

The Problem: General Relativity in Terms of Geographical Maps

$$ds^2 = \sum_{\mu\nu} g_{\mu\nu}\ dx_\mu dx_\upsilon$$

- metric field: device for extracting measurable distances ds between any two nearby points from their coordinate differences dx_ν
- gravitational field: physical field that have certain values at a certain point with coordinates x_ν, just like the electromagnetic field .

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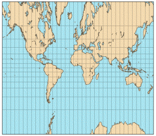

Figure: cylindrical projection

$$ds^2 = 4\left(1 + \frac{x^2}{R^2} + \frac{y^2}{R^2}\right)^{-2} dx^2 + 4\left(1 + \frac{x^2}{R^2} + \frac{y^2}{R^2}\right)^{-2} dy^2$$

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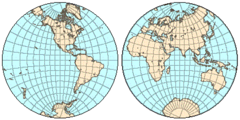

Figure: stereographic projection

$$ds^2 = \left(1 - \frac{y^2}{R^2}\right) dx^2 + \left(1 - \frac{y^2}{R^2}\right)^{-1} dy^2$$

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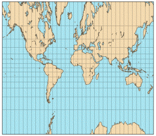

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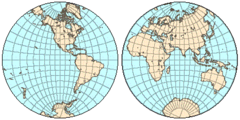

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$g_{\mu\nu}$ = map legend

- the treasure is buried at $x = 45, y = 85$ in open ocean
- the numbers x, y are meaningless without the map legend $g_{\mu\nu}$

$g_{\mu\nu}$ = map legend = treasure

- the treasure is the value of $g_{\mu\nu}$ at $x = 45, y = 85$ in open space
- $g_{\mu\nu}$ different values at the same point $x = 45, y = 85$ in different map legends

'values of the $g_{\mu\nu}$ at x, y' = meaningful statement?

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Early relativists thought in terms of geographical maps, rather than of the geometry of the surface

- problem: the Greenwich meridian intersects the equator at the point $x = 45, y = 85$ in one map but not in the other
- solution: $x = 45, y = 85$ is not the *same* physical point, the same physical point is where Greenwich meridian intersects the equator

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intersections $\Longrightarrow$ coincidences

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The Problem: General Relativity in Terms of Geographical Maps

- classical mechanics $\Longrightarrow$ ‘the same place in different times’ meaningless
- special relativity $\Longrightarrow$ ‘at the same time in different places’ meaningless
- general relativity $\Longrightarrow$ at the same place at the same time meaningless

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… rethinking the very notion of where something happens

The Proto-PCA: The Physical Meaning of Coordinates

Die Relativitäts-Theorie. [1])

Von

A. Einstein in Prag.

> “In mathematical physics, it is customary to relate things to coordinate systems [. . .] What is essential in this relating-to-something is the following: when we state anything whatsoever about the location of a point, we always indicate the coincidence of this point with some point of a specific other physical system. If, for example, I choose myself as this material point, and say, ‘I am at this location in this hall,’ then I have brought myself into spatial coincidence with a certain point of this hall, or rather, I have asserted this coincidence. This is done in mathematical physics by using three numbers, the so-called coordinates, to indicate with which points of the rigid system, called the coordinate system, the point whose location is to be described coincides.”

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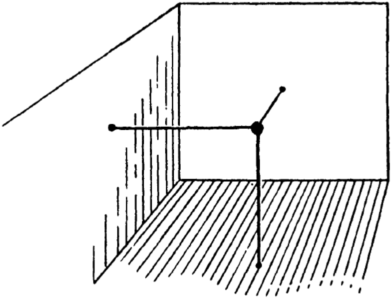
In classical mechanics and special relativity ‘where’ something occurs were always meant as a physical coincidence with some material body of reference



(the surface of the Earth, the city of Zurich, or the walls of the hall in which Einstein was giving his lecture)

The Proto-PCA: The Physical Meaning of Coordinates

- a good non-accelerating massive cubical scaffolding of rigidly connected measuring rods and clocks $K(x, y, z, t)$
  - the readings of rods and clocks at rest with respect to K directly give the numbers x, y, z, t.
  - the components of $\vec{E}(x, y, z, t)$ and $\vec{B}(x, y, z, t)$ with respect to K can be determined by observing the paths of test particles with respect to K.
  - the paths of test particles are successive encounters with certain points of K when the handles of a clock are on a certain position on the dial.
- theoretical asymmetry between a good acceleration-free $K(x, y, z, t)$ and the bad coordinate system $K'(x', y', z', t')$ accelerating with respect to the former

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Einstein, 1912–1914

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2. x, y, z, t as readings on rods and clock: espr. $\vec{E}(x, y, z, t) \rightarrow$ has a physical meaning

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the universal nature of gravitation
made the interpretation (1) impossible.

It becomes impossible to establish whether K is a 'good' coordinate system in which nonaccelerated rods and clocks at rest reliably read coordinate differences or a 'bad' one K' accelerating in the opposite direction, in which coordinate numbers do not directly denote distances.

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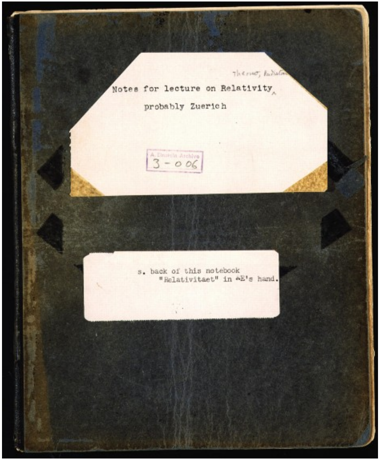

Figure: Zurich Notebook

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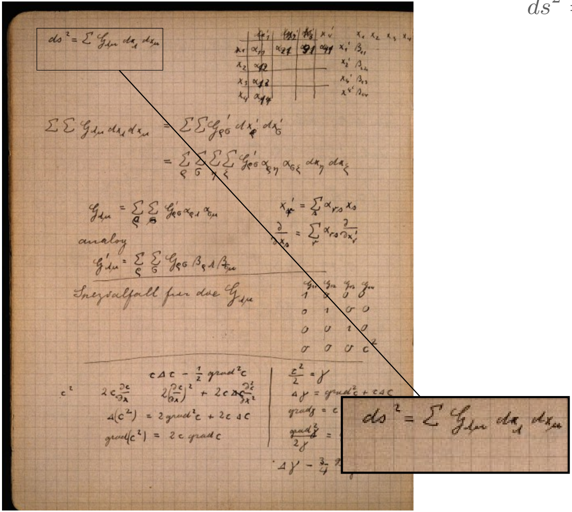

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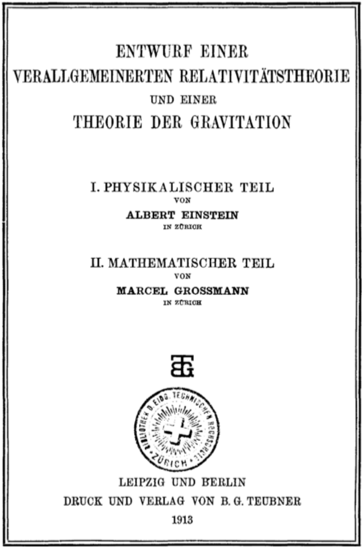
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IN ZÜRICH

II. MATHEMATISCHER TEIL
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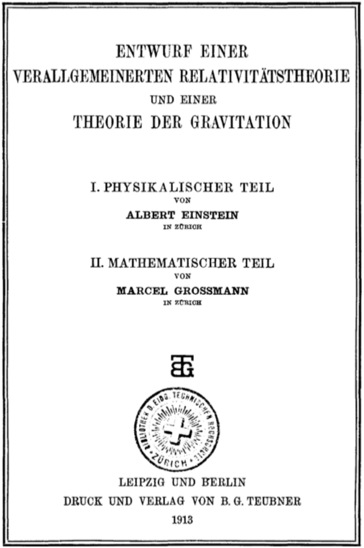

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DRUCK UND VERLAG VON B. G. TEUBNER

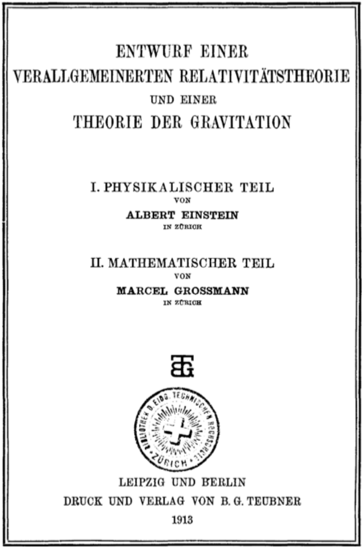
1913

$$ds^2 = \sum_{\mu\nu} g_{\mu\nu}\, dx_\mu dx_\upsilon$$

- metric field: extract measurable distances ds between any two nearby points from their coordinate differences dx_ν
- gravitational field : physical field that have certain values at a certain point with coordinates x_1, x_2, x_3, x_4, just like the electromagnetic field.

$$g_{\mu\nu}(x_1, x_2, x_3, x_4)$$

The *Entwurf* Theory



ENTWURF EINER
VERALLGEMEINERTEN RELATIVITÄTSTHEORIE
UND EINER
THEORIE DER GRAVITATION

I. PHYSIKALISCHER TEIL
VON
ALBERT EINSTEIN
IN ZÜRICH

II. MATHEMATISCHER TEIL
VON
MARCEL GROSSMANN
IN ZÜRICH

LEIPZIG UND BERLIN
DRUCK UND VERLAG VON B. G. TEUBNER
1913

“ . . . there cannot exist relationships between the spacetime coordinates x_1, x_2, x_3, x_4 and the results of measurements obtainable by means of measuring rods and clocks that would be as simple as those in the old relativity theory ”

labels $\neq$ distances

The *Entwurf* Theory

Den Tensor $\Theta_{\mu\nu}$ nennen wir den (kontravarianten) Spannungs-Energietensor der materiellen Strömung. Der Gleichung (10) schreiben wir einen Gültigkeitsbereich zu, der über den speziellen Fall
[23] der Strömung inkohärenter Massen weit hinausgeht. Die Gleichung stellt allgemein die Energiebilanz zwischen dem Gravitationsfelde und einem beliebigen materiellen Vorgang dar; nur ist für $\Theta_{\mu\nu}$ der dem jeweilen betrachteten materiellen System entsprechende Spannungs-Energietensor einzusetzen. Die erste Summe in der Gleichung enthält die örtlichen Ableitungen der Spannungen bzw. Energiestromdichte und die zeitlichen Ableitungen der Impuls- bzw. Energiedichte; die zweite Summe ist ein Ausdruck für die Wirkungen, welche vom Schwerefelde auf den materiellen Vorgang übertragen werden.

§ 5. Die Differentialgleichungen des Gravitationsfeldes.

Nachdem wir die Impuls-Energiegleichung für die materiellen Vorgänge (mechanische, elektrische und andere Vorgänge) mit bezug auf das Gravitationsfeld aufgestellt haben, bleibt uns noch folgende Aufgabe. Es sei der Tensor $\Theta_{\mu\nu}$ für den materiellen Vorgang gegeben. Welches sind die Differentialgleichungen, welche die Größen g_{ik}, d. h. das Schwerefeld zu bestimmen gestatten? Wir suchen mit anderen Worten die Verallgemeinerung der Poissonschen Gleichung

$$\Delta\varphi = 4\pi k \varrho.$$

Zur Lösung dieser Aufgabe haben wir keine so vollkommen zwangläufige Methode gefunden, wie für die Lösung des vorhin behandelten Problems. Es war nötig, einige Annahmen einzuführen, deren Richtigkeit zwar plausibel erscheint, aber doch nicht evident ist.

Die gesuchte Verallgemeinerung wird wohl von der Form sein

$$\varkappa \cdot \Theta_{\mu\nu} = \Gamma_{\mu\nu}, \qquad (11)$$

wo $\varkappa$ eine Konstante, $\Gamma_{\mu\nu}$ ein kontravarianter Tensor zweiten Ranges ist, der durch Differentialoperationen aus dem Fundamentaltensor $g_{\mu\nu}$ hervorgeht. Dem Newton-Poissonschen Gesetz entsprechend wird man geneigt sein zu fordern, daß diese Gleichungen (11) zweiter Ordnung sein sollen. Es muß aber hervorgehoben werden, daß es sich als unmöglich erweist, unter dieser Voraussetzung einen Differentialausdruck $\Gamma_{\mu\nu}$ zu finden, der eine Verallgemeinerung von $\Delta\varphi$ ist, und sich beliebigen Transformationen gegenüber als Tensor erweist.[1]) A priori kann allerdings nicht in Abrede gestellt werden, daß die endgültigen, genauen Gleichungen der Gravitation von höherer als zweiter Ordnung sein könnten. Es besteht daher immer noch die Möglichkeit, daß die

1) Vgl. II. Teil, § 4, Nr. 2.

field equations relating the source variables (charge density, mass energy distribution, etc.) to the field variables (gravitational field, electromagnetic field, etc.)

$$\Delta\varphi = 4\pi k\varrho$$

$$\kappa\Theta_{\mu\nu} = \Gamma_{\mu\nu}$$

- I want to know the the values of $g_{\mu\nu}$ at a point x_1, x_2, x_3, x_4
- The $g_{\mu\nu}$ tell me where the point x_1, x_2, x_3, x_4

Besso-Einstein's Argument against General Covariance

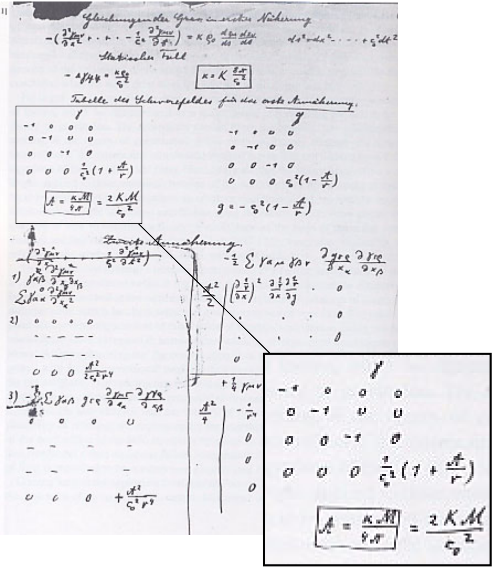


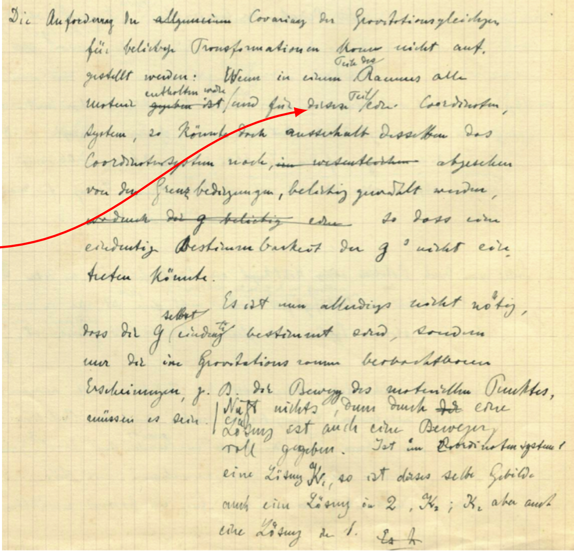
Figure: Einstein-Besso Manuscript — June 1913

Entwurf-theory field equations of limited covariance $\rightarrow$ Mercury's anomalous perihelion precession

“ Is the static gravitational field $g_{\mu\nu} = 1, 1$ to 3, $g_{44} = f(x, y, z)$ a particular solution? Or is it the general solution expressed in particular coordinates?[a] ”

Besso-Einstein's Argument against General Covariance

[Besso:] The requirement of [general] covariance of the gravitational equations under arbitrary transformations cannot be imposed: if all matter [is given] were contained in one part of space and for this part of space a coordinate system [is given], then, outside of it, the coordinate system, except for boundary conditions be chosen arbitrarily, so that a unique determinability of the g's cannot be obtained



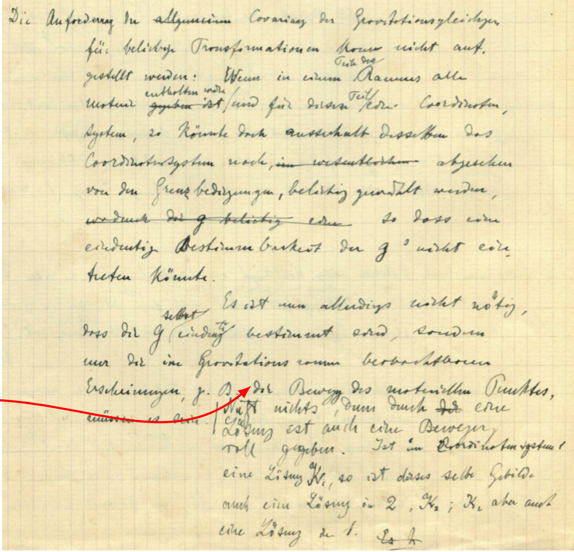
Die Anforderung der allgemeinen Covarianz der Gravitationsgleichungen für beliebige Transformationen kann nicht aufgestellt werden: Wenn in einem Teile des Raumes alle Materie enthalten wäre und für diesen Teil ein Coordinaten-System, so könnte doch ausserhalb desselben das Coordinatensystem noch, abgesehen von den Grenzbedingungen, beliebig gewählt werden, so dass eine eindeutige Bestimmbarkeit der g nicht eintreten könnte.

Es ist nun allerdings nicht nötig, dass die g selbst eindeutig bestimmt sind, sondern nur die im Gravitationsraum beobachtbaren Erscheinungen, z. B. die Bewegung des materiellen Punktes, müssen es sein. | Nützt nichts, denn durch eine Lösung ist auch eine Bewegung voll gegeben. Ist im Koordinatensystem 1 eine Lösung K_1, so ist dieses selbe Gebilde auch eine Lösung in 2, K_2; K_2 aber auch eine Lösung in 1. Es h

Figure: Besso-Memo – August 1913

Besso-Einstein’s Argument against General Covariance

[Besso:] It is, however, not necessary that the themselves are determined uniquely, only the observable phenomena in the gravitation space, e.g., the motion of a material point, must be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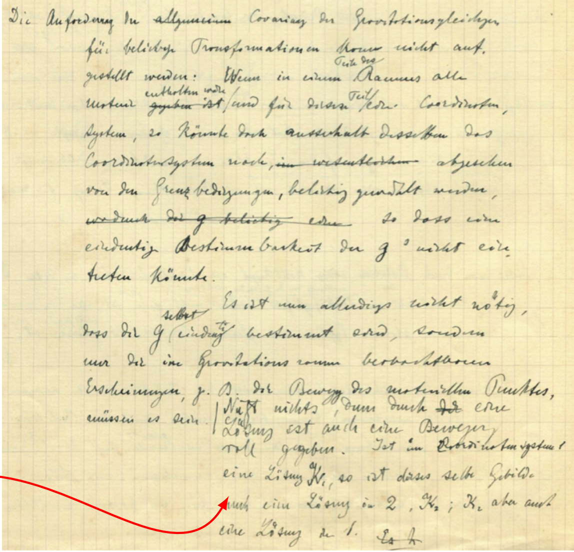
Besso-Einstein's Argument against General Covariance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auch eine Lösung in 2, K_2; K_2 aber auch
eine Lösung in 1. Es ist

[Einstein:] Of no use, since with a solution a motion is also fully given. If in coordinate system 1, there is a solution K_1, then this same construct is also a solution in 2, K_2; K_2 however, also a solution in 1

Besso-Einstein’s Argument against General Covariance

2) In den letzten Tagen fand ich den Beweis dafür, daß eine derartige allgemein kovariante Lösung überhaupt nicht existieren kann.

herab. Das ganze Problem der Gravitation wäre also befriedigend gelöst, wenn es auch gelänge, bezüglich beliebiger Substitutionen kovariante Gleichungen zu finden, welchen die das Gravitationsfeld selbst bestimmenden Größen $g_{\mu\nu}$ genügen. In dieser Weise gelang es uns aber nicht, das Problem zu lösen[2]). Die Lösung gelang aber in der Weise, daß nachträglich wieder das Bezugssysten spezialisiert wurde. Zu diesem Wege gelangt man ungezwungen durch folgende Überlegung. Es ist klar, daß für irgend-

1) Kottler, Über die Raumzeitlinien der Minkowskischen Welt. Wien. Ber. 121, 1912.

2) In den letzten Tagen fand ich den Beweis dafür, daß eine derartige allgemein kovariante Lösung überhaupt nicht existieren kann.

> “ In the last days I found a proof that such a generally covariant solution to the problem cannot exist at all ”

Einstein, 1913.

Besso-Einstein's Argument against General Covariance

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> “ a unique determination of the $g_{\mu\nu}$ out of the $T_{\mu\nu}$ is impossible (rigorously provable) ”
>
> *Einstein to Ehrenfest, Nov. 11, 1913*

Besso-Einstein's Argument against General Covariance

Es gebe in unserer vierdimensionalen Mannigfaltigkeit einen Teil L, in welchem ein „materieller Vorgang" nicht stattfinde, in welchem also die $\Theta_{\mu\nu}$ verschwinden. Durch die außerhalb L gegebenen $\Theta_{\mu\nu}$ sind gemäß unserer Annahme überall, also auch im Innern von L die $\gamma_{\mu\nu}$ vollkommen bestimmt. Wir denken uns nun statt der ursprünglichen Koordinaten x_ν neue Koordinaten x'_ν eingeführt von folgender Art. Außerhalb L sei überall $x_\nu = x'_\nu$; innerhalb L aber sei wenigstens für einen Teil von L und wenigstens für einen Index ν $x_\nu \neq x'_\nu$. Es ist klar, daß durch eine derartige Substitution erreicht werden kann, daß wenigstens für einen Teil von L $\gamma'_{\mu\nu} \neq \gamma_{\mu\nu}$ ist. Andererseits ist überall $\Theta'_{\mu\nu} = \Theta_{\mu\nu}$, nämlich außerhalb L, weil für dieses Gebiet $x'_\nu = x_\nu$ ist, innerhalb L aber, weil für dies Gebiet $\Theta_{\mu\nu} = 0 = \Theta'_{\mu\nu}$ ist. Hieraus folgt, daß in dem betrachteten Falle, wenn alle Substitutionen als berechtigte zugelassen werden, zu dem nämlichen System der $\Theta_{\mu\nu}$ mehr als ein System der $\gamma_{\mu\nu}$ gehört.

Wenn also — wie dies in der Arbeit geschehen ist — an der Forderung festgehalten wird, daß durch die $\Theta_{\mu\nu}$ die $\gamma_{\mu\nu}$ vollständig bestimmt sein sollen, so ist man genötigt, die Wahl des Bezugssystems einzuschränken. Diese Einschränkung wird in unserer Arbeit dadurch erzielt, daß für den materiellen Vorgang und das Gravitationsfeld zusammen die Gültigkeit der Erhaltungssätze, d. h. die Gültigkeit von vier Gleich… Aus diese… tionsfeldes…

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Figure: Appendix to Einstein and Grossmann, 1914

Einstein inverted Besso's argument

mass surrounded by empty space → region L without any material process surrounded by matter

Besso-Einstein's Argument against General Covariance

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Aus dieser[…]
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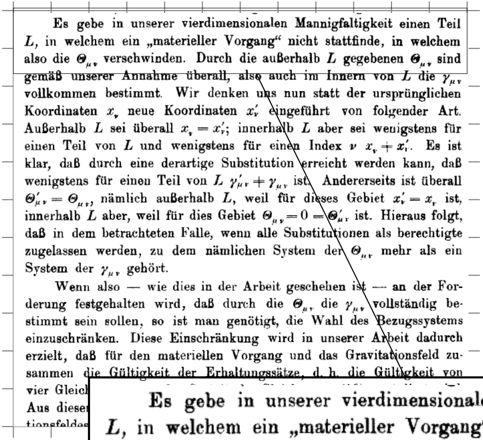
Besso-Einstein's Argument against General Covariance



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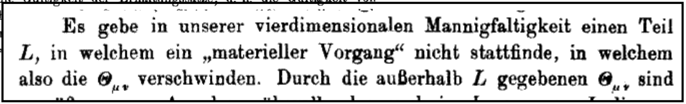

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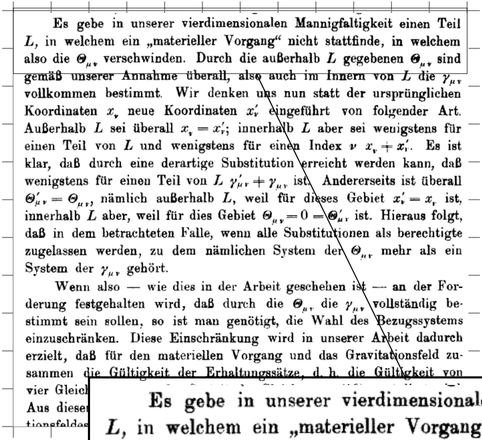
Besso-Einstein's Argument against General Covariance



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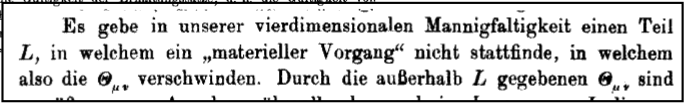

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Besso-Einstein's Argument against General Covariance

a) Wenn das Bezugssystem ganz willkürlich gewählt wird, dann können die $g_{\mu\nu}$ durch die $\mathfrak{T}_{\sigma\nu}$ überhaupt nicht vollständig bestimmt sein. Man denke sich nämlich die $\mathfrak{T}_{\sigma\nu}$ und $g_{\sigma\nu}$ überall gegeben, und es mögen in einem Teil Φ des vierdimensionalen Raumes alle $\mathfrak{T}_{\sigma\nu}$ verschwinden. Ich kann nun ein neues Bezugssystem einführen, welches außerhalb Φ mit dem ursprünglichen vollkommen übereinstimmt, innerhalb Φ aber (ohne Verletzung der Stetigkeit) von ihm verschieden ist. Bezieht man nun alles auf dieses neue Bezugssystem, wobei die Materie durch $\mathfrak{T}_{\sigma\nu}'$, das Gravitationsfeld durch $g_{\mu\nu}'$ ausgedrückt wird, so ist zwar überall

$$\mathfrak{T}_{\sigma\nu}' = \mathfrak{T}_{\sigma\nu},$$

dagegen werden im Innern von Φ die Gleichungen

$$g_{\mu\nu}' = g_{\mu\nu}$$

sicherlich nicht alle erfüllt sein[1]). Hieraus folgt die Behauptung.

Will man erzielen, daß eine vollständige Bestimmung der $g_{\mu\nu}$ (Gravitationsfeld) aus den $\mathfrak{T}_{\sigma\nu}$ (Materie) möglich sei, so kann dies nur durch Beschränkung in der Wahl des Bezugssystems erreicht werden.

1) Die Gleichungen sind so zu verstehen, daß auf den linken Seiten jeweilen den unabhängigen Variabeln x_ν' dieselben Zahlenwerte erteilt werden wie auf den rechten Seiten den Variabeln x_ν.

$$T'_{\mu\nu} = T_{\mu\nu}$$

⇓

$$g'_{\mu\nu} \neq g_{\mu\nu}$$

Besso-Einstein’s Argument against General Covariance

1) Die Gleichungen sind so zu verstehen, daß auf den linken Seiten jeweilen den unabhängigen Variabeln x_ν' dieselben Zahlenwerte erteilt werden wie auf den rechten Seiten den Variabeln x_ν.

a) Wenn da… kürlich gewäh… $g_{\mu\nu}$ durch di… ständig bestim… nämlich die $\mathfrak{T}_{\sigma\nu}$ und $g_{\sigma\nu}$ überall gegeben, und es mögen in einem Teil Φ des vierdimensionalen Raumes alle $\mathfrak{T}_{\sigma\nu}$ verschwinden. Ich kann nun ein neues Bezugssystem einführen, welches außerhalb Φ mit dem ursprünglichen vollkommen übereinstimmt, innerhalb Φ aber (ohne Verletzung der Stetigkeit) von ihm verschieden ist. Bezieht man nun alles auf dieses neue Bezugssystem, wobei die Materie durch $\mathfrak{T}_{\sigma\nu}'$, das Gravitationsfeld durch $g_{\mu\nu}'$ ausgedrückt wird, so ist zwar überall

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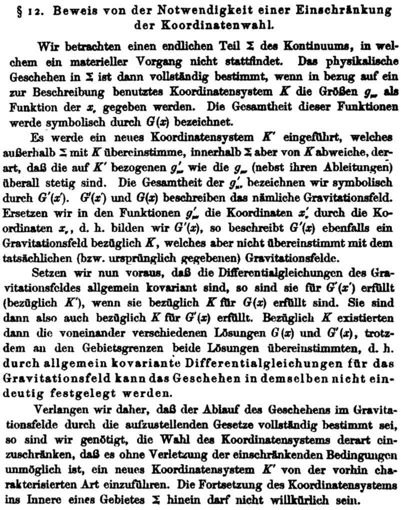
“ . . . the same numerical values are always assigned to the independent variables x_ν on the left sides as to the variables x'_ν on the right sides ”

The §12 Argument



§ 12. Beweis von der Notwendigkeit einer Einschränkung der Koordinatenwahl.

Wir betrachten einen endlichen Teil Σ des Kontinuums, in welchem ein materieller Vorgang nicht stattfindet. Das physikalische Geschehen in Σ ist dann vollständig bestimmt, wenn in bezug auf ein zur Beschreibung benutztes Koordinatensystem K die Größen $g_{\mu\nu}$ als Funktion der x_ν gegeben werden. Die Gesamtheit dieser Funktionen werde symbolisch durch $G(x)$ bezeichnet.

Es werde ein neues Koordinatensystem K' eingeführt, welches außerhalb Σ mit K übereinstimme, innerhalb Σ aber von K abweiche, derart, daß die auf K' bezogenen $g'_{\mu\nu}$ wie die $g_{\mu\nu}$ (nebst ihren Ableitungen) überall stetig sind. Die Gesamtheit der $g'_{\mu\nu}$ bezeichnen wir symbolisch durch $G'(x')$. $G'(x')$ und $G(x)$ beschreiben das nämliche Gravitationsfeld. Ersetzen wir in den Funktionen $g'_{\mu\nu}$ die Koordinaten x'_ν durch die Koordinaten x_ν, d. h. bilden wir $G'(x)$, so beschreibt $G'(x)$ ebenfalls ein Gravitationsfeld bezüglich K, welches aber nicht übereinstimmt mit dem tatsächlichen (bzw. ursprünglich gegebenen) Gravitationsfelde.

Setzen wir nun voraus, daß die Differentialgleichungen des Gravitationsfeldes allgemein kovariant sind, so sind sie für $G'(x')$ erfüllt (bezüglich K'), wenn sie bezüglich K für $G(x)$ erfüllt sind. Sie sind dann also auch bezüglich K für $G'(x)$ erfüllt. Bezüglich K existierten dann die voneinander verschiedenen Lösungen $G(x)$ und $G'(x)$, trotzdem an den Gebietsgrenzen beide Lösungen übereinstimmten, d. h. durch allgemein kovariante Differentialgleichungen für das Gravitationsfeld kann das Geschehen in demselben nicht eindeutig festgelegt werden.

Verlangen wir daher, daß der Ablauf des Geschehens im Gravitationsfelde durch die aufzustellenden Gesetze vollständig bestimmt sei, so sind wir genötigt, die Wahl des Koordinatensystems derart einzuschränken, daß es ohne Verletzung der einschränkenden Bedingungen unmöglich ist, ein neues Koordinatensystem K' von der vorhin charakterisierten Art einzuführen. Die Fortsetzung des Koordinatensystems ins Innere eines Gebietes Σ hinein darf nicht willkürlich sein.

Figure: Einstein, 1914

§12

region Σ of spacetime

$$G(x) \to G'(x)' \to G'(x)$$

“There are then two different solutions $G(x)$ and $G'(x)$ relative to K, even though the solutions coincide on the boundary of the domain Σ”

Back to General Covariance

Zur allgemeinen Relativitätstheorie.

Von A. Einstein.

In den letzten Jahren war ich bemüht, auf die Voraussetzung der Relativität auch nicht gleichförmiger Bewegungen eine allgemeine Relativitätstheorie zu gründen. Ich glaubte in der Tat, das einzige Gravitationsgesetz gefunden zu haben, das dem sinngemäß gefaßten, allgemeinen Relativitätspostulate entspricht, und suchte die Notwendigkeit gerade dieser Lösung in einer im vorigen Jahre in diesen Sitzungsberichten erschienenen Arbeit[1] darzutun.

Eine erneute Kritik zeigte mir, daß sich jene Notwendigkeit auf dem dort eingeschlagenen Wege absolut nicht erweisen läßt; daß dies doch der Fall zu sein schien, beruhte auf Irrtum. Das Postulat der Relativität, soweit ich es dort gefordert habe, ist stets erfüllt, wenn man das Hamiltonsche Prinzip zugrunde legt; es liefert aber in Wahrheit keine Handhabe für eine Ermittelung der Hamiltonschen Funktion H des Gravitationsfeldes. In der Tat drückt die die Wahl von H einschränkende Gleichung (77) a. a. O. nichts anderes aus, als daß H eine Invariante bezüglich linearer Transformationen sein soll, welche Forderung mit der der Relativität der Beschleunigung nichts zu schaffen hat. Ferner wird die durch Gleichung (78) a. a. O. getroffene Wahl durch Gleichung (77) keineswegs festgelegt.

Aus diesen Gründen verlor ich das Vertrauen zu den von mir aufgestellten Feldgleichungen vollständig und suchte nach einem Wege, der die Möglichkeiten in einer natürlichen Weise einschränkte. So gelangte ich zu der Forderung einer allgemeineren Kovarianz der Feldgleichungen zurück, von der ich vor drei Jahren, als ich zusammen mit meinem Freunde Grossmann arbeitete, nur mit schwerem Herzen abgegangen war. In der Tat waren wir damals der im nachfolgenden gegebenen Lösung des Problems bereits ganz nahe gekommen.

Wie die spezielle Relativitätstheorie auf das Postulat gegründet ist, daß ihre Gleichungen bezüglich linearer, orthogonaler Transfor-

[1] Die formale Grundlage der allgemeinen Relativitätstheorie. Sitzungsberichte XLI, 1914, S. 1066—1077. Im folgenden werden Gleichungen dieser Abhandlungen beim Zitieren durch den Zusatz »a. a. O.« von solchen der vorliegenden Arbeit unterschieden.

four papers in November 1915 ...

Back to General Covariance

“ A point mass, the sun, is located at the origin of the coordinate system. The gravitational field this point mass produces can be calculated from the [field equations] [. . .] Nevertheless, we should consider that the $g_{\mu\nu}$ are still not completely determined mathematically by [the field] equations [. . .] Yet we are justified in assuming that all these solutions can be reduced to one another by such transformations that they are distinguished (by the given boundary conditions) formally but not, however, physically, from one another ”

Einstein, 1915

Back to General Covariance

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Back to General Covariance

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Back to General Covariance

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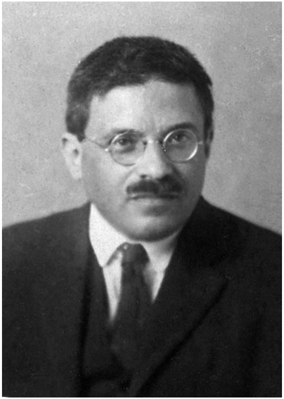
Back to General Covariance



> "In 1914 Einstein developed an almost philosophical proof [against general covariance] (I mean e.g. §12 of his paper of 19.XI.1914. Is this proof correct?"
>
> *Ehrenfest to Lorentz, Dec. 23, 1915*

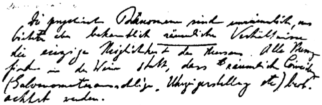
Back to General Covariance

“ What is observed here — if we neglect, at first, all direct metrical evaluations — is only the complete or partial spatiotemporal coincidence [*Zusammenfallen*] or non-coincidence [*Nichtzusammenfallen*] of parts of the measuring instrument with parts of the measured object ”

Kretschammn1915, *12/21/1915*

Deckung gebrachten Teile des Meßgegenstandes. Beobachtet wird hierbei — wenn wir zunächst von allen direkten Größenschätzungen absehen — nur das völlig oder teilweise erreichte räumlich-zeitliche Zusammenfallen oder Nichtzusammenfallen von Teilen des Meßinstrumentes mit Teilen des Meßgegenstandes. Oder allgemein: rein topologische Beziehungen zwischen

Erich Kretschmann


Die physikalischen Phänomene sind [illegible], es bilden aber bekanntlich räumliche Verhältnisse die einzige Möglichkeit der Messung. Alle Messungen finden in der Weise statt, dass räumliche Coincidenzen (Galvanometerausschläge, Uhrzeigerstellung etc) beobachtet werden.

Moritz Schlick

> “All measurements happen in a way that spatial coincidences [*Coinzidenzen*] (Galvanometer deflections, positions on a clock dial etc.) can be observed ... (Zurich 1910)”

Private PCA: The Einstein-Ehrenfest Correspondence

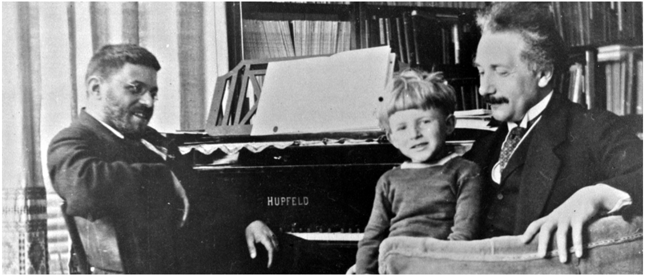


Figure: Einstein-Ehrenfest Correspondence Dec. 1915–Jan. 1916

Private PCA: The Einstein-Ehrenfest Correspondence

“ In §12 of my paper of last year, everything is correct [. . .] A contradiction to the uniqueness of the event does not follow at all from the fact that both systems $G(x)$ and $G'(x)$, related to the same frame of reference, satisfy the conditions of the grav. field. [. . .]

1. the reference system has no real meaning
2. that the (simultaneous) materialization of two different g systems [. . .] within the same area of the continuum is [. . .] impossible

In place of §12 the following consideration must appear. Whatever is physically real in events in the universe (as opposed to that which is dependent on the choice of a reference system) consist in spatio-temporal coincidences*

*and in nothing else! ”

Ehrenfest to Einstein, Dec. 26, 1916

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In place of §12 the following consideration must appear. Whatever is physically real in events in the universe (as opposed to that which is dependent on the choice of a reference system) consist in spatio-temporal coincidences* [. . .] [Two inter-transformable $g_{\mu\nu}$-systems] are entirely equivalent

*and in nothing else!

"

Ehrenfest to Einstein, Dec. 26, 1916

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In place of §12 the following consideration must appear. Whatever is physically real in events in the universe (as opposed to that which is dependent on the choice of a reference system) consist in spatio-temporal coincidences* This is because they have in common all the spatial-temporal point coincidences, that is, all the observables

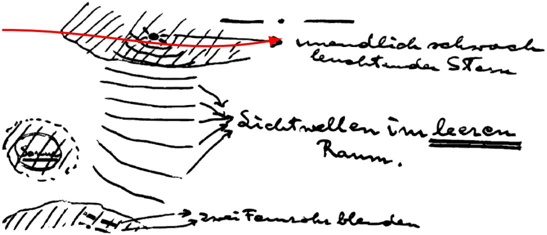
*and in nothing else!

”

Ehrenfest to Einstein, Dec. 26, 1916

> “ I will defend the philosophy of §12 against your refutation ”
>
> *Ehrenfest to Einstein, Jan. 1, 1916*

star emitting infinitely weak light



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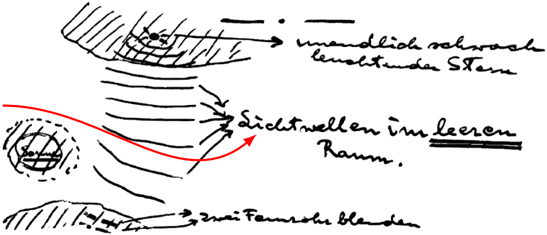
light waves in empty space



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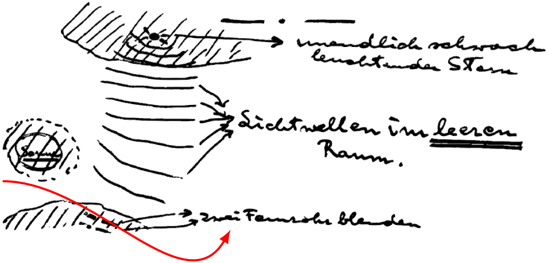
an aperture, two telescopes a photographic plate



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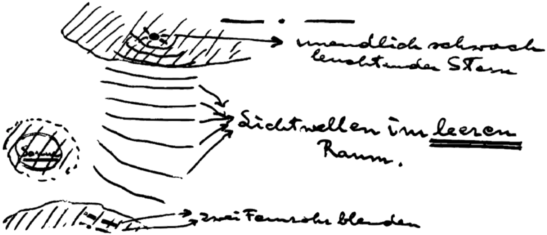
The situation described using $x_1 \ldots x_4, T_{\mu\nu}, g_{\mu\nu}$, etc.



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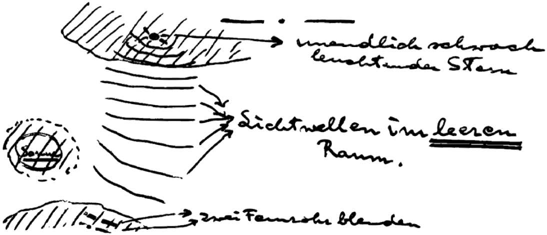

> “Bravo the photrogr. plate is darkened (a true ‘coincidence explanation’).”

Private PCA: The Einstein-Ehrenfest Correspondence

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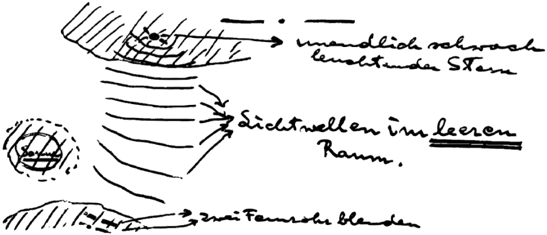

> “Then let’s see the philosophy of §12 at work $G(x) \to G'(x') \to G'(x)$”

Private PCA: The Einstein-Ehrenfest Correspondence

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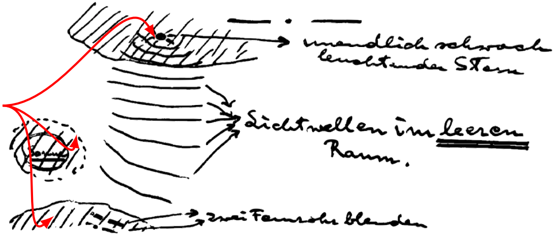
In the cross-hatched region occupied by matter, the coordinate system is kept fix



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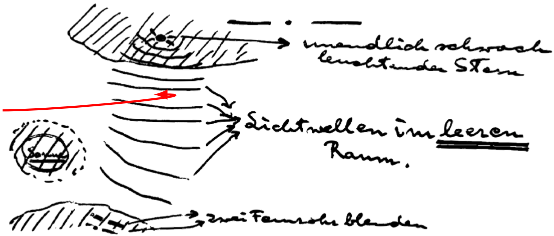
Private PCA: The Einstein-Ehrenfest Correspondence

“ I will defend the philosophy of §12 against your refutation ”

Ehrenfest to Einstein, Jan. 1, 1916

whereas in the non-cross-hatched empty space the latter is strongly changed



“ Then let’s see the philosophy of §12 at work $G(x) \to G'(x') \to G'(x)$ ”

Private PCA: The Einstein-Ehrenfest Correspondence

“ You look at me laughing quietly and you say ‘go ahead, young friend, and describe if you like, the empty space

with old coordinates x
and brand new $G'(x)$

nothing observable, no ‘coincidence’ would change!? Now I’m astonished and *angry* over your laugh and

I claim with clenched fist:

‘*If with the old* x *and the old* $G(x)$ *and the new* $G'(x)$ *one calculates the darkening of the plate, then one should calculate the non-darkening of the plate with old* x *and new* $G'(x)$’ ”

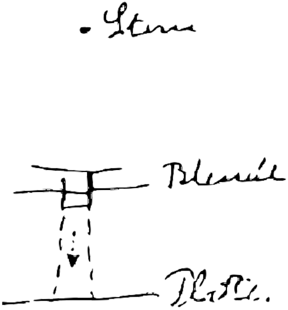
Ehrenfest to Einstein, Jan. 1, 1916

Private PCA: The Einstein-Ehrenfest Correspondence

> “The root of your difficulty lies in the fact that you instinctively treat the reference system as something real”

Einstein to Ehrenfest, Jan. 5, 1916

- draw star, the aperture and the plate onto completely deformable tracing paper

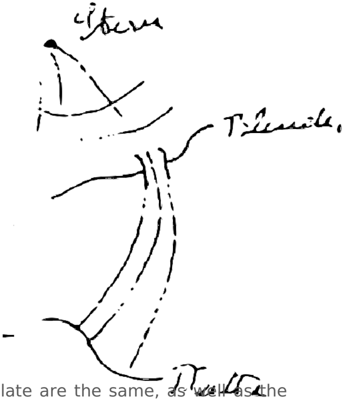


Private PCA: The Einstein-Ehrenfest Correspondence

> "The root of your difficulty lies in the fact that you instinctively treat the reference system as something real"

Einstein to Ehrenfest, Jan. 5, 1916

- deform the tracing paper arbitrarily in the plane of the paper[a]
- make another tracing on the letter paper



[a] the coordinates of the star, the aperture and the plate are the same, as well as the boundary conditions at infinity

Private PCA: The Einstein-Ehrenfest Correspondence

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Einstein to Ehrenfest, Jan. 5, 1916

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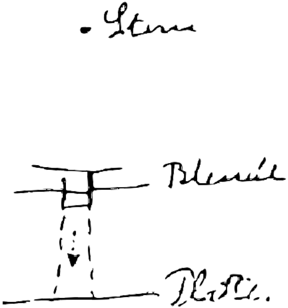

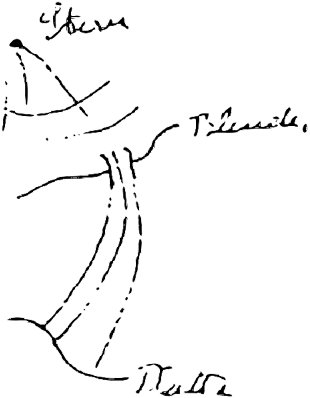

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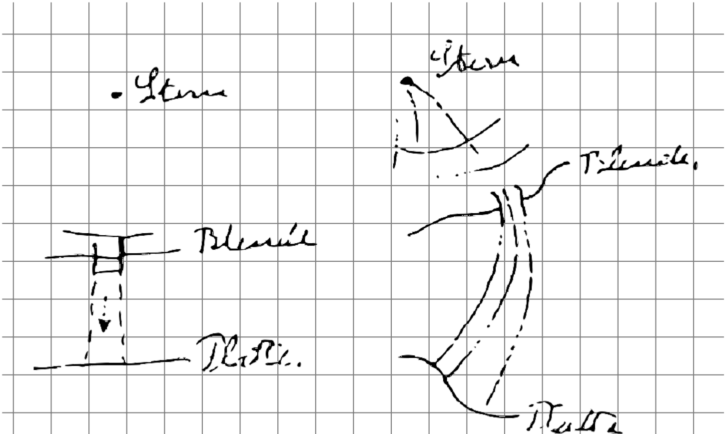

Private PCA: The Einstein-Ehrenfest Correspondence

“When you relate the figure once again to the orthogonal writing paper coordinates, the solution is mathematically different from the original, and naturally also with respect to the $g_{\mu\nu}$. But physically it is exactly the same, since the writing paper coordinate system is only something imaginary. Always the same points are illuminated on the plate. [. . .] As long as the drawing paper, i.e., ‘the space’, is unreal, both diagrams do not differ at all. Always the same points are illuminated on the plate [. . .] It all depends on coincidences, e.g., whether the plate points are hit by the light or not”

Einstein to Ehrenfest, Jan. 5, 1916

- The same diagram’s lines do not intersect at the same point of the latter paper.
- The same point is there where the same diagram worldlines intersect.

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- The same point is there where the same diagram worldlines intersect.

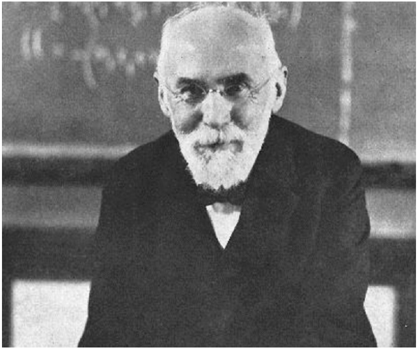
Private PCA: The Einstein-Ehrenfest Correspondence

> "When you relate the figure once again to the orthogonal writing paper coordinates, the solution is mathematically different from the original, and naturally also with respect to the $g_{\mu\nu}$. But physically it is exactly the same, since the writing paper coordinate system is only something imaginary. Always the same points are illuminated on the plate. [. . .] As long as the drawing paper, i.e., 'the space', is unreal, both diagrams do not differ at all. Always the same points are illuminated on the plate [. . .] It all depends on coincidences, e.g., whether the plate points are hit by the light or not"

Einstein to Ehrenfest, Jan. 5, 1916

- The same diagram's lines do not intersect at the same point of the latter paper.
- The same point is there where the same diagram worldlines intersect.

Private PCA: The Lorentz-Ehrenfest Correspondence



†

Lorentz-Ehrenfest Correspondence, January 1916*

† Thanks to A.J. Cox!.

Private PCA: The Lorentz-Ehrenfest Correspondence

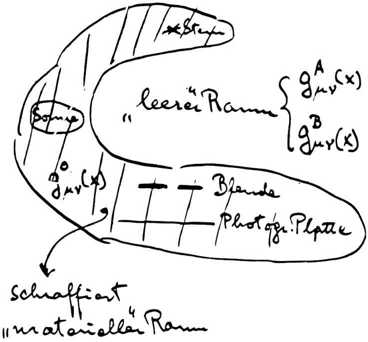
‘matter-free’ field:

- $g_{\mu\nu} = F(x_\alpha)$ (field I)
- $g'_{\mu\nu} = F'(x'_\alpha)$
- $F'(x_\alpha)$ (field II)

> “ This is a new solution, differing from the first. [...] (I) and (II) now in fact differ physically, since in field (I) a material point moves uniformly along a straight line, whereas one can easily see that this is not the ease in field (II). [...] Of course there exists a similar indeterminacy also in other cases, e.g. the sun’s field, calculated by Einstein. ”
>
> *Lorentz to Ehrenfest, Jan. 9, 1916*

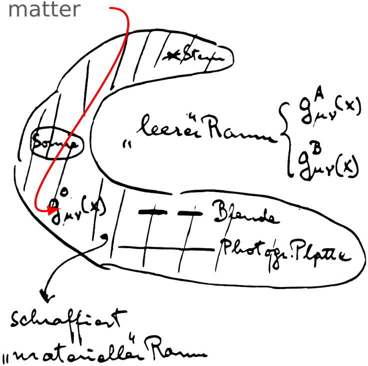
Private PCA: The Lorentz-Ehrenfest Correspondence

x_1, x_2, x_3, x_4

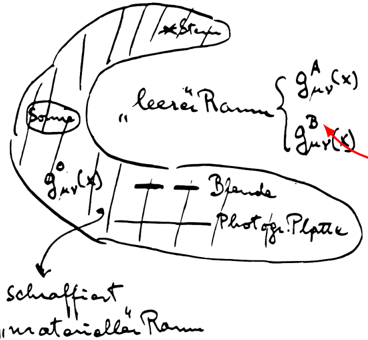


Private PCA: The Lorentz-Ehrenfest Correspondence

$g^0_{\mu\nu}(x)$ in cross-hatched matter region



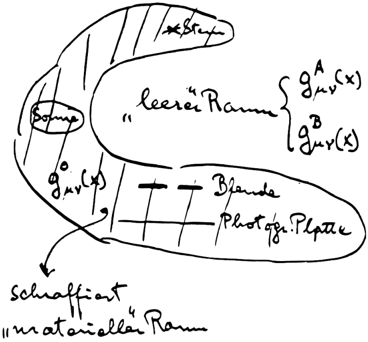
x_1, x_2, x_3, x_4



x_1, x_2, x_3, x_4

$g^A_{\mu\nu}(x)$ and $g^B_{\mu\nu}(x)$ in empty space

Private PCA: The Lorentz-Ehrenfest Correspondence

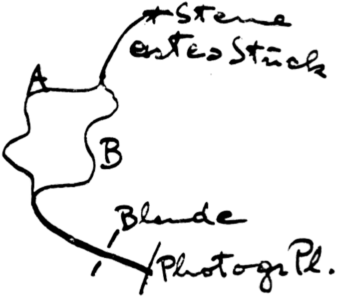


$$x_1, x_2, x_3, x_4$$

$$ds = 0$$

- light rays traverse different x_1, x_2, x_3, x_4 values in empty space
- light rays meet the same point x_1, x_2, x_3, x_4 on the plate in matter region

Private PCA: The Lorentz-Ehrenfest Correspondence



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- light rays meet the same point x_1, x_2, x_3, x_4 on the plate in matter region

> “Einstein is fully correct when he says: A and B produce the same darkening of the photographic plate”

Ehrenfest to Lorentz, Jan. 9, 1916

Private PCA: The Lorentz-Ehrenfest Correspondence

"I admit that we observe only 'coincidences'. [...] In connection with this I now realize that in the two fields I and II that I spoke of in my last letter, it is true that the separate phenomena do not take the same course in relation to the coordinate-system, but that in both of them the coincidences do occur in the same way. I was too much a prisoner of the idea that our equations must fully reproduce the relations between the phenomena and the chosen coordinate-system, whereas we can be happy if they duly reproduce the mutual relations between the phenomena."

Lorentz to Ehrenfest, Jan. 10, 1916

Private PCA: The Lorentz-Ehrenfest Correspondence

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Private PCA: The Lorentz-Ehrenfest Correspondence

'Jupiter will be at φ, ϑ, r at the time t.'

Private PCA: The Lorentz-Ehrenfest Correspondence

'Jupiter will be at φ, ϑ, r at the time t.'

- The claim that a planet is at certain point at a certain instant respect to the sun means that the astronomical sight lines up the planet and a fixed star.
- the trajectory of a light ray joining the star and the orbit of the planet, passing through a telescope co-moving with earth, and living marks on a photographic plate is interrupted by the passage of Venus when the handles of a clock is on a certain position on the dial.
- any statement about a position of the planet is ultimately a statement about the intersections among the worldline of the light ray coming from the star, the worldline of the planet, and that of the telescope, etc..
- statement about the intersection of worldlines reduce to statement about coincidences

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- The question whether these worldlines will intersect at the same spacetime location φ, ϑ, r, t according to all inter-trasfromable $g_{\mu\nu}$ is meaningless
- the same spacetime location is where the same worldline intersect, which does not depend on the $g_{\mu\nu}$ used

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Private PCA: The Einstein-Lorentz Correspondence

Figure: . . . Prachtswinkel auf diesem öden Planeten

“ I see that you have thought over the theory entirely and have familiarized yourself with the idea that all of our experiences in physics refer to coincidences ”

Einstein to Lorentz, Jan. 17, 1916

Public PCA: Einstein 1916 Review Article

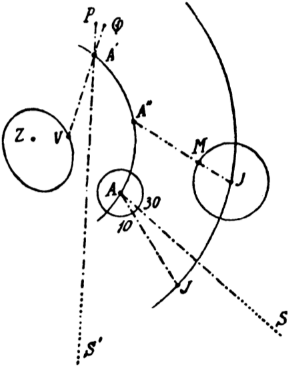
Physics. – *"On* EINSTEIN'S *Theory of gravitation."* I. By Prof. H. A. LORENTZ.

(Communicated in the meeting of February 26, 1916).

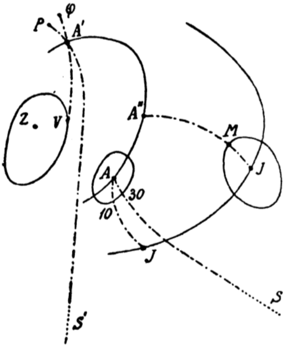
Now EINSTEIN has made the striking remark [3]) that the only thing we can learn from our observations and with which our theories are essentially concerned, is the existence of these coincidences.

[3]) In a correspondence I had with him.

Veldfiguur I.



Veldfiguur II.



Public PCA: Einstein 1916 Review Article

1916. № 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. *Die Grundlage der allgemeinen Relativitätstheorie;*

von A. Einstein.

Public PCA: Einstein 1916 Review Article

> "That this requirement of general co-variance, which takes away from space and time the last remnant of physical materiality [*Gegenständlichkeit*], is a natural one, will be seen from the following reflexion. All our spacetime assessments [*Konstatierungen*] invariably amount to a determination of spacetime coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings [*Begegnungen*] of two or more of these points. Moreover, the results of our measurings are nothing but assessments [*Konstatierungen*] of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial"

Einstein, 1916

Public PCA: Einstein 1916 Review Article

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Einstein, 1916

Public PCA: Einstein 1916 Review Article

- coordinates x_1, x_2, x_3, x_4 do not tell us where such coincidences happen with respect to a given coordinate system;
- the four numbers x_1, x_2, x_3, x_4 are only a bookkeeping system to keep track of which coincidences we are referring to in a given coordinate system.

The PCA in Einstein's Popular Book on Relativity

> "When we were describing the motion of a material point relative to a body of reference, we stated nothing more than the encounters of this point with particular points of the reference-body. [...] in conjunction with the observation of the encounter of the hands of clocks with particular points on the dials. [...] The following statements hold generally: Every physical description [...] refers to the spacetime coincidence."
>
> *Einstein, 1917, 1916*

- pre-GR: the material point coincide with a point of a scaffolding
- GR: coincidence of trajectories of two points

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Reception of The PCA: Schlick and Kretschmann

- Moritz Schlick: the PCA is essential → the choice between different spacetimes which agree on point-coincidences is conventional[a]
- Erich Kretschmann: the PCA argument is trivial → the choince of the coordinate system to describe the same spacetime is conventional since all coordinate system agree on point-coincidences[b]

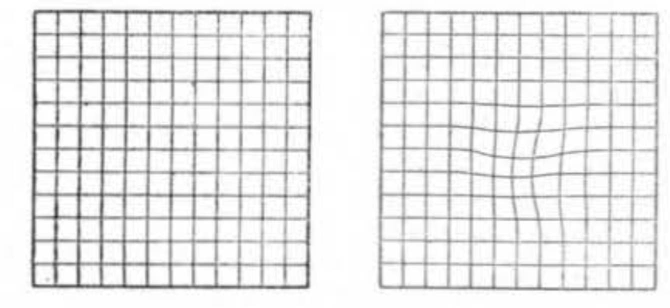
[a] Schlick, 1917.
[b] Kretschmann, 1917.

Return of the PCA. The Correspondence with Gustav Mie

flapping rod paradox

- a rod at rest in one coordinate system.
- If one uses light rays as a standard, the rod is appears to be straight.
- introducing a fast changing gravitational field, the rod would appear flapping.

Return of the PCA. The Correspondence with Gustav Mie



The two coordinate systems are indeed geometrically equivalent, however, physically, the first coordinate system is clearly better than the second one

Return of the PCA. The Correspondence with Gustav Mie

“ I do not agree at all with your reflection about the bent (flapping) rod. All physical descriptions that yield the same observable relations (coincidences) are equivalent in principle, provided that both descriptions are also based on the same laws of nature. The choice of coordinates can have great practical importance from the point of view of clarity of description; in principle, though, it is entirely insignificant. It means nothing that ‘arbitrary gravitational fields’ occur, depending on the coordinate choice; [. . .] in principle, one can actually only learn something about the latter by eliminating the coordinates. The ghost of absolute space haunts your rod example. ”

Einstein to Mie, Feb. 8, 1918

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Last Use of the PCA. The Relativity Principle

918. № 4.

ANNALEN DER PHYSIK

VIERTE FOLGE. BAND 55.

a) *Relativitätsprinzip:* Die Naturgesetze sind nur Aussagen
er zeiträumliche Koinzidenzen; sie finden deshalb ihren einzig
türlichen Ausdruck in allgemein kovarianten Gleichungen.

Prinzipielles zur allgemeinen Relativitätstheorie;
von A. Einstein.

Eine Reihe von Publikationen der letzten Zeit, insbesondere e neulich in diesen Annalen 53. Heft 16 erschienene scharf-nnige Arbeit von Kretschmann, veranlassen mich, noch-als auf die Grundlagen der allgemeinen Relativitätstheorie rückzukommen. Dabei ist es mein Ziel, lediglich die Grund-danken herauszuheben, wobei ich die Theorie als bekannt raussetze.

Die Theorie, wie sie mir heute vorschwebt, beruht auf drei auptgesichtspunkten, die allerdings keineswegs voneinander abhängig sind. Sie seien im folgenden kurz angeführt und arakterisiert und hierauf im nachfolgenden von einigen Seiten leuchtet:

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c) *Machsches Prinzip*[1]: Das *G*-Feld ist *restlos* durch die assen der Körper bestimmt. Da Masse und Energie nach

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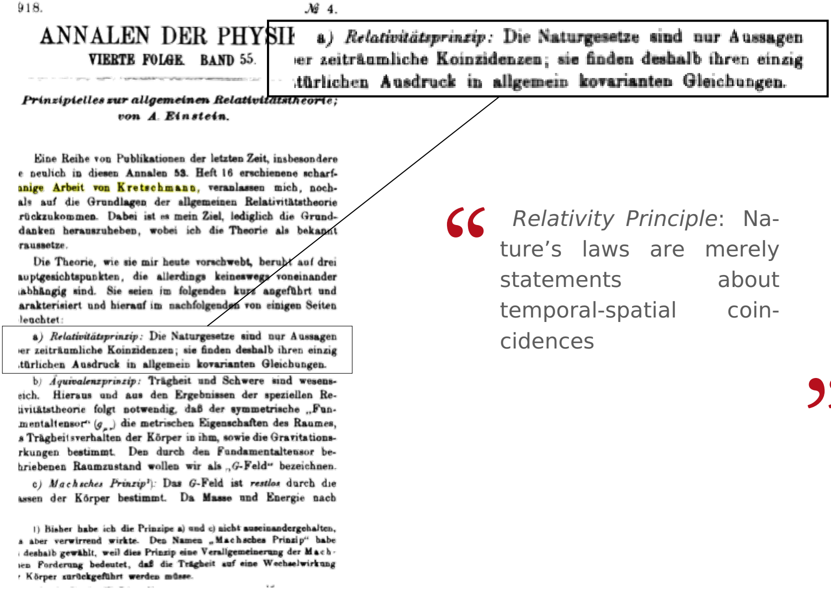

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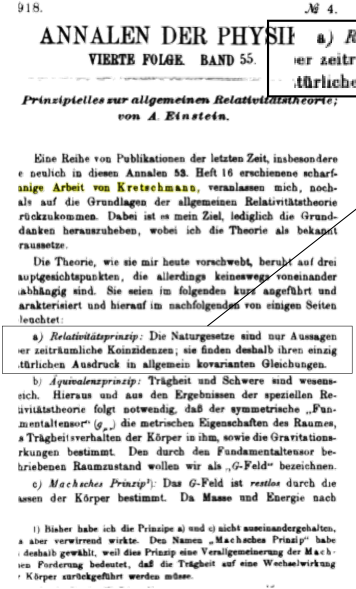
> “ *Relativity Principle*: Nature’s laws are merely statements about temporal-spatial coincidences ”

Last Use of the PCA. The Relativity Principle



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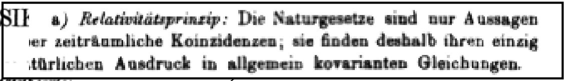
assen der Körper bestimmt. Da Masse und Energie nach

1) Bisher habe ich die Prinzipe a) und c) nicht auseinandergehalten,
s aber verwirrend wirkte. Den Namen „Machsches Prinzip" habe
deshalb gewählt, weil dies Prinzip eine Verallgemeinerung der Mach-
ien Forderung bedeutet, daß die Trägheit auf eine Wechselwirkung
r Körper zurückgeführt werden müsse.



a) *Relativitätsprinzip:* Die Naturgesetze sind nur Aussagen
ber zeiträumliche Koinzidenzen; sie finden deshalb ihren einzig
atürlichen Ausdruck in allgemein kovarianten Gleichungen.

- any theory can be trivially presented in a way in which coordinates are meaningless parameters.
- pre-general-relativistic theories can also be written in terms of coordinates that are reading on rods and clocks

Last Use of the PCA. The Relativity Principle

918. № 4.

ANNALEN DER PHYSIK

VIERTE FOLGE. BAND 55.

Prinzipielles zur allgemeinen Relativitätstheorie;
von A. Einstein.

Eine Reihe von Publikationen der letzten Zeit, insbesondere
e neulich in diesen Annalen 53. Heft 16 erschienene scharf-
nnige Arbeit von Kretschmann, veranlassen mich, noch-
als auf die Grundlagen der allgemeinen Relativitätstheorie
rückzukommen. Dabei ist es mein Ziel, lediglich die Grund-
danken herauszuheben, wobei ich die Theorie als bekannt
raussetze.

Die Theorie, wie sie mir heute vorschwebt, beruht auf drei
auptgesichtspunkten, die allerdings keineswegs voneinander
abhängig sind. Sie seien im folgenden kurz angeführt und
arakterisiert und hierauf im nachfolgenden von einigen Seiten
leuchtet:

a) *Relativitätsprinzip:* Die Naturgesetze sind nur Aussagen
er zeiträumliche Koinzidenzen; sie finden deshalb ihren einzig
türlichen Ausdruck in allgemein kovarianten Gleichungen.

a) *Relativitätsprinzip:* Die Naturgesetze sind nur Aussagen
er zeiträumliche Koinzidenzen; sie finden deshalb ihren einzig
türlichen Ausdruck in allgemein kovarianten Gleichungen.

b) *Äquivalenzprinzip:* Trägheit und Schwere sind wesens-
eich. Hieraus und aus den Ergebnissen der speziellen Re-
tivitätstheorie folgt notwendig, daß der symmetrische „Fun-
mentaltensor" ($g_{\mu\nu}$) die metrischen Eigenschaften des Raumes,
s Trägheitsverhalten der Körper in ihm, sowie die Gravitations-
rkungen bestimmt. Den durch den Fundamentaltensor be-
hriebenen Raumzustand wollen wir als „G-Feld" bezeichnen.

c) *Machsches Prinzip*[1]: Das G-Feld ist *restlos* durch die
assen der Körper bestimmt. Da Masse und Energie nach

1) Bisher habe ich die Prinzipe a) und c) nicht auseinandergehalten,
s aber verwirrend wirkte. Den Namen „Machsches Prinzip" habe
deshalb gewählt, weil dies Prinzip eine Verallgemeinerung der Mach-
en Forderung bedeutet, daß die Trägheit auf eine Wechselwirkung
r Körper zurückgeführt werden müsse.

- pre-general-relativistic theories contain not just statements about coincidences but also statements about coordinate systems which serve for their description
- general relativity entails nothing more than relations between coincidences, the statements of which are independent of the choice of coordinates.

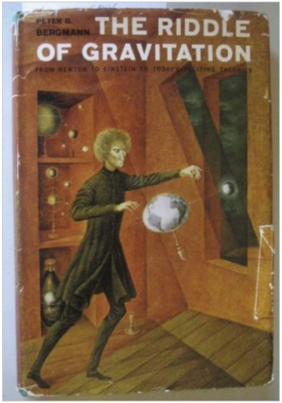
Last Use of the PCA. The Relativity Principle

> “ Coordinates are not an adequate means of identifying world points physically. A physically meaningful statement about events is necessarily a statement about coincidences; that is, the prototype statement should be : ‘Events A, B, C, ... took place at the same world point’* ”

- the value of the $g^{ij}(x^i)$ is not an observable
- the value $A^i(x^i)$ at a point is not an observable
- the coincidence $g^{ij}(A^i)$ is an observables (in principle can be measured)

*Bergmann and Komar, 1962, see also Bergmann, 1961, 1962.

Conclusion



> What is left of the world-point? [...] It is not inconceivable that in the present formulation of general relativity the world point continues to exist as a relic from previous physical theories, to be discarded at the next stage of theoretical development ...[a]

[a] Bergmann, *The Riddle of Gravitation*, 1968.

Thanks!

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