Appearance and Reality

Einstein and the Early Debate on the Reality of Length Contraction

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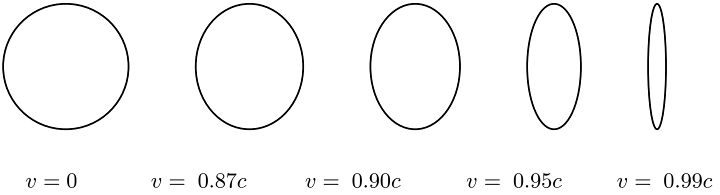
Introduction: Length Contraction



length contraction: all bodies contract in the direction of their motion by a factor $l = l_0\sqrt{1 - \frac{v^2}{c^2}}$

- **Lorentz contraction**: dynamical phenomenon due to the action of molecular forces
- **Einstein contraction**: kinematical phenomenon due to the definition of simultaneity

Part I

Bell and the CERN Debate on the Reality of Length Contraction

Bell and the CERN Debate

Collected papers on quantum philosophy

Speakable and unspeakable in quantum mechanics

J. S. BELL
CERN

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How to teach special relativity

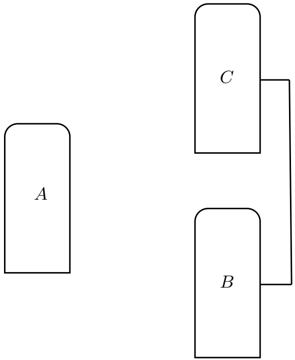
I have for long thought that if I had the opportunity to teach this subject, I would emphasize the continuity with earlier ideas. Usually it is the discontinuity which is stressed, the radical break with more primitive notions of space and time. Often the result is to destroy completely the confidence of the student in perfectly sound and useful concepts already acquired[1].

If you doubt this, then you might try the experiment of confronting your students with the following situation[2]. Three small spaceships, A, B, and C, drift freely in a region of space remote from other matter, without rotation and without relative motion, with B and C equidistant from A (Fig. 1).

On reception of a signal from A the motors of B and C are ignited and they accelerate gently[3] (Fig. 2).

Let ships B and C be identical, and have identical acceleration programmes. Then (as reckoned by an observer in A) they will have at every moment the same velocity, and so remain displaced one from the other by a fixed distance. Suppose that a fragile thread is tied initially between projections from B and C (Fig. 3). If it is just long enough to span the required distance initially, then as the rockets speed up, it will become too short, because of its need to Fitzgerald contract, and must finally break. It must break when, at a sufficiently high velocity, the artificial prevention of the natural contraction imposes intolerable stress.

- identical spaceships B and C with identical acceleration programs: same velocity with respect to A and fixed distance
- a fragile thread is tied initially between B and C.



Prediction of special relativity: stresses and **the cable breaks!**

Bell and the CERN Debate

> “Is it really so? This old problem came up for discussion once in the CERN canteen. A distinguished experimental physicist refused to accept that the thread would break and regarded my assertion, that indeed it would, as a personal misinterpretation of special relativity. We decided to appeal to the CERN Theory Division for arbitration and made a (not very systematic) canvas of opinion in it. There emerged a clear consensus that the thread would **not** break!”

Bell 1987

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- Einstein contraction $\Longrightarrow$ **apparent**.
- Lorentz contraction $\Longrightarrow$ **real**

> “Of course, many people who give this wrong answer at first get the right answer on further reflection. Usually, they feel obliged to work out how. It is only after working this out, and perhaps only with a residual feeling of unease, that such people finally accept a conclusion that is perfectly trivial in terms of A's account of things, including the [Lorentz] contraction. It is my impression that those with a more classical education, knowing something of the reasoning of [...] Lorentz [...], as well as that of Einstein, have stronger and sounder instincts.”
>
> *Bell 1987*

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Lorentzian pedagogy vs. Lorentzian philosophy

Today Philosophical Debate

Brown (2005) and Brown and Pooley (2001, 2006) and many others:

- **theoretical claim**: we should adopt Lorentzian philosophy
- **historical claim**: Einstein shared the Lorentzian philosophy

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dynamical interpretation of SR

This paper:

- **theoretical claim**: is a non sequitur
- **historical claim**: is false

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kinematical interpretation of SR

Part II

The Early Debate on The Reality of The Length Contraction: Outline

- **Born** (1909) put forward a relativistic definition of a rigid body for the case of linear acceleration.
- **Ehrenfest** (1909) argued that a Born rigid cylinder could not be given angular acceleration (it would ultimately break!).
- **Herglotz** (1910) and **Noether** (1910) proved that a Born rigid body has only three degrees of freedom instead of the six that a classical rigid body has.

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crisis: the old concept of a rigid body had to be abandoned, and nothing suitable could be found to replace it

The Early Debate on The Reality of The Length Contraction: Outline

- **Ignatowski** (1910) and **Varićak** (1911) tried to defend relativity in the *Physikalische Zeitschrift*
  - Ehrenfest's argument is comprehensible from the point of view of Lorentz's theory, in which contraction is a real phenomenon.
  - Ehrenfest's argument is at odds with Einstein's theory, in which the contraction is only apparent and results from an arbitrary choice of clock synchronization.

- **Ehrenfest** (1910) and **Einstein** (1911) replied in the *Physikalische Zeitschrift*
  - relativistic length contraction is a kinematic effect and, in this sense, is only apparent.
  - nevertheless, length contraction is real, since it can in principle be ascertained empirically.

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Ehrenfest's thought experiment illustrates this point:
stress effects would not manifest if the old kinematics held.

Physikalische Zeitschrift vs. CERN Canteen

- **Einstein-Ehrenfest vs. Ignatowski-Varićak** on the pages of the *Physikalische Zeitschrift* in the 1910s
  - Ehrenfest paradox $\rightarrow$Born rigid rotating disc: intolerable stresses and breakage.
- **Bell vs. colleagues** at the tables of the CERN canteen in the 1970s
  - Bell paradox $\rightarrow$Born rigid thread-between-spaceships: intolerable stresses and breakage.

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- **Einstein**: the emergence of relativistic stresses demonstrates that length contraction is a kinematic effect, and nevertheless it is real.
- **Bell**: the emergence of relativistic stresses demonstrates that length contraction is a real effect, and therefore it requires a dynamic explanation.

Physikalische Zeitschrift vs. CERN Canteen

- **Varićak** and **Ignatowski**: kinematic = apparent and dynamic = real.
    - Bell and CERN physicists = Varićak and Ignatowski.
- **Einstein** and **Ehrenfest**: contraction is apparent because it disappears for the comoving observer, but it is also real because it can be ‘experimentally verified’.

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Philosophy within the history of physics.

Part III

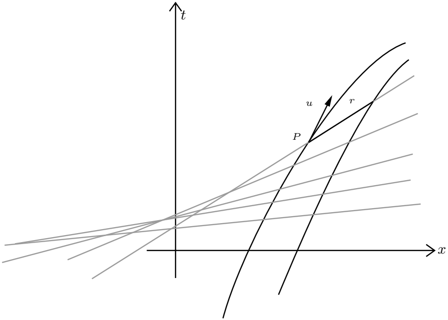
Born, Ehrenfest and the Relativistic Rigid Body

Born’s Definition of Rigidity

Versammlung deutscher Naturforscher und Ärzte zu Salzburg

- on September 21, 1909, a young **Einstein** delivered his first major address at a scientific meeting arguing for a new theory of radiation.
- the following day, September 22, 1909, **Born** presented a simplified, geometrical version of his relativistic definition of rigidity.

Born's Definition of Rigidity



- a bar in linear acceleration along the x-axis moves rigidly if its length r is **constant** as measured in its successive inertial rest frames.
- Born's definition captures the intuitive notion that a rigid body in motion remains **free of strains** because its changing acceleration maintains the body's rest-frame dimensions.
- in relativity theory, every part of the bar must undergo **different** rates of acceleration to remain rigid.
- thus, rigidity is not a property of a rigid body but a **program** that involves applying forces

Born's Definition of Rigidity

- Einstein immediately pointed out after the talk that a Born rigid body at rest could never be brought into **uniform rotation**.
- since a non-accelerated **Cartesian coordinate system** is nothing but a rigid body, Einstein wondered how such a reference body would behave when accelerating.

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> "The treatment of the uniformly rotating rigid body seems to me of great importance because of an extension of the relativity principle to uniformly rotating systems according to analogous trains of thought as I did in the last § on the uniformly accelerated translation of my paper for the *Zeitschr[ift] f[ür] Radioaktivit*"

(Einstein to Sommerfeld, Sep. 29, 1909; CPAE, Vol. 1, Doc. 179).

würde sehr interessant sein, zu sehen, ob man einen einmal begonnenen Kristall dazu bringen könnte, seine Ansicht über seine „Konstante" zu ändern, wenn man die Konzentration oder die Temperatur der Mutterflüssigkeit, in der er sich bildet, verändert.

East Hampton, Long Island, im Juli 1909.

(Nach dem Manuskript aus dem Englischen übersetzt von Max Iklé.)

(Eingegangen 13. September 1909.)

Gleichförmige Rotation starrer Körper und Relativitätstheorie.

Von P. Ehrenfest.

Bei dem Versuch, die Kinematik relativ-starrer Körper von der gleichförmigen, geradlinigen Translation auf beliebige Bewegungen zu verallgemeinern, gelangt man im Anschluß an Minkowskis Ideen zu folgendem Ansatz:

Ein Körper verhält sich relativ-starr, heißt: er deformiert sich bei einer beliebigen Bewegung fortlaufend so, daß jedes seiner infinitesimalen Elemente in jedem Moment für einen ruhenden Beobachter gerade diejenige Lorentz-Kontraktion (gegenüber dem Ruhezustand) aufweist, welche der Momentan-Geschwindigkeit des Element-Mittelpunktes entspricht.

Als ich mir vor einiger Zeit die Konsequenzen dieses Ansatzes veranschaulichen wollte, stieß ich auf Folgerungen, die zu zeigen scheinen, daß obiger Ansatz schon für einige sehr einfache Bewegungstypen zu Widersprüchen führt.

Nun hat Herr Born in einer kürzlich erschienenen Arbeit[1]) eine Definition der Relativ-Starrheit gegeben, die alle überhaupt möglichen Bewegungen umfaßt. Herr Born hat diese Definition — dem Grundgedanken der Relativitätstheorie entsprechend — nicht auf das Maßsystem eines ruhenden Beobachters basiert, sondern auf die (Minkowskischen) Maßbestimmungen sozusagen eines Kontinuums von infinitesimalen Beobachtern, die mit den Punkten des ungleichförmig bewegten Körpers mitwandern: jedem von ihnen soll in seinem Maß seine infinitesimale Umgebung dauernd undeformiert erscheinen.

Beide Definitionen der Relativ-Starrheit sind aber — wenn ich richtig verstanden habe — äquivalent. — Es sei deshalb gestattet, kurz auf den einfachsten Typus einer Bewegung hinzuweisen, bei dem die erstangeführte Definition schon zu Widersprüchen führt: die gleichförmige Rotation um eine feste Achse.

In der Tat: Es sei gegeben ein relativ-starrer Zylinder vom Radius R und der Höhe H. Es werde ihm allmählich eine schließlich konstant bleibende Drehbewegung um seine Achse erteilt. Sei R' der Radius, den er bei dieser Bewegung für einen ruhenden Beobachter aufweist. Dann müßte R' zwei einander widersprechende Forderungen erfüllen:

a) Die Peripherie des Zylinders muß gegenüber dem Ruhezustand eine Kontraktion zeigen:

$$2\pi R' < 2\pi R,$$

denn jedes Element der Peripherie bewegt sich in seiner eigenen Richtung mit der Momentangeschwindigkeit $R'\omega$.

b) Betrachtet man irgendein Element eines Radius, so steht seine Momentangeschwindigkeit normal zu seiner Erstreckung; also können die Elemente eines Radius gegenüber dem Ruhezustand keinerlei Kontraktion aufweisen. Es müßte sein:

$$R' = R.$$

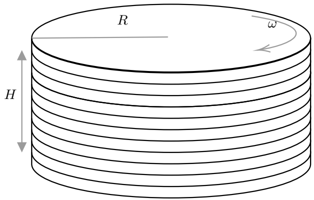
Bemerkung: Will man die Deformation jedes Elementes nicht nur von der Momentangeschwindigkeit des Elementmittelpunktes abhängen lassen, sondern auch noch von der momentanen Rotationsgeschwindigkeit des Elementes, so muß die Deformationsfunktion außer der Lichtgeschwindigkeit c noch eine universelle, dimensionierte Konstante enthalten oder es müssen in sie auch noch Beschleunigungen des Elementmittelpunktes eingehen.

St. Petersburg, Sept. 1909.

(Eingegangen 29. September 1909.)

1) M. Born, Die Theorie des starren Elektrons in der Kinematik des Relativitäts-Prinzipes. Ann. d. Phys. **30**, 1, 1909. Vergl. auch diese Zeitschr. **10**, 814, 1909.

Ehrenfest's Born Rigid Rotating Cylinder



- Born rigid cylinder of radius R and height H rotating with angular velocity $\vec{\omega} < \dfrac{c}{R}$.
- rim line elements are Lorentz contracted by $\sqrt{1 - \dfrac{v^2}{c^2}}$, but not R.
- Circumference U measured from the radius: $U = 2\pi R$.
- Circumference U measured from the rim: $U < 2\pi R$.

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Contradiction!

Ehrenfest's Born Rigid Rotating Cylinder

- Ehrenfest's friend, **Herglotz** (1910)
- A student of Sommerfeld, **Noether** (1910).

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Rigid body has only three degrees of freedom.

Part IV

Einstein and Varićak

- **Ehrenfest** believed that his thought experiment should have convinced physicists to abandon the Lorentz-Einstein theory altogether—the ballistic theory of light developed by Ritz (1908).
- **Varićak** attempted to come in support of relativity and circumvent Ehrenfest's objection.
- **Einstein** sided with Ehrenfest and embraced his result as yet another example of the unfeasibility of the notion of a rigid body in relativity theory.

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> "It is certain that Ehrenfest's consideration is incomplete. But at its core, he is right."

(Einstein to Varićak, Apr. 23, 1910; CPAE, Vol. 5[10], Doc. 202b)

Part V

The Ehrenfest-Ignatowski Debate

The Ehrenfest-Ignatowski Debate

- Debate between **Born, Planck, Abraham**: “Planck’s remark cannot be seen as a weakening of Ehrenfest’s objection” (Abraham 1910, 531)
- Around July 1910, **Ignatowski** likely sent Einstein the manuscript of a paper on relativistic rigid bodies to obtain his blessing (Ignatowski 1910): “With Ignatowski it is indeed as you suspected. He draws conclusions that contradict mine and then wants my approval” (Einstein to Hopf, Aug. 19, 1910; CPAE, Vol. 5, Doc. 221).
- Ignatowski became aware of Ehrenfest’s work by reading a paper by **Stead** (July 1910), in which the cylinder was replaced by a disk with an elastic membrane that can bend without resistance.

Ignatowski's Objection

"To my mind, the whole thing seems to be a misunderstanding. Let's measure a line element along the circumference of the disk synchronously and sum over the circumference; we get a value smaller than $2\pi R$, where R means the radius of the disk. There is absolutely no contradiction in this, but everything is explained by the definition of a synchronous measurement. Mr. Ehrenfest's objection is nothing more than a confirmation that a uniform rotation satisfies the condition [of Born rigidity], and that accordingly a line element along the circumference of the cylinder measured synchronously appears shortened. In general, we can determine the true [*wahre*] form and dimension of a rigid body by measurement when and only when the body is at rest. Measurements on moving bodies yield only apparent [*scheinbar*] values."

(Ignatowski 1910, 630)

Zu Herrn v. Ignatowskys Behandlung der Bornschen Starrheitsdefinition.

Von Paul Ehrenfest.

§ 1. Ein Körper, welcher der Bornschen Relativ-Starrheits-Forderung[1]) genügt, kann auf keine Weise aus der Ruhe in den Zustand einer gleichförmigen Rotation um eine feste Achse übergeführt werden. — Dieses Theorem wurde so durchsichtig und bündig bewiesen[2]), daß es von allen

1) M. Born, Ann. d. Phys. **30**, 1, 1909.

2) P. Ehrenfest, diese Zeitschr. **10**, 918, 1909; G. Herglotz, Ann. d. Phys. **31**, 393, 1910; F. Noether, Ann. d. Phys. **31**, 919, 1910.

Zu Herrn v. Ignatowskys Behandlung der Bornschen Starrheitsdefinition. II[1]).

Von P. Ehrenfest.

Zur größeren Deutlichkeit erlaube ich mir folgende Konstatierungen.

1. Kann ein Körper, welcher der Bornschen

1) I: diese Zeitschr. 11, 1127, 1910.

Ehrenfest's Reply

A circular **disk** with equally spaced marks over its radius and circumference is provided (Ehrenfest 1910, 1129):

- An observer B at rest relative to the disk records these marks of the stationary disk on a piece of tracing paper P.
- While the disk is rotating, at the moment his clock points to t, the stationary observer B holds a piece of tracing paper P_1 over it and traces all the marks on the rotating disk.
- The stationary observer B measures the mark distribution on the stationary tracing images Π and Π_1 on a piece of paper at rest.

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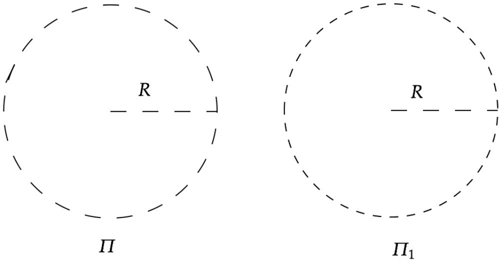
Π_1 is Ignatowski's 'simultaneously measured' by the stationary observer at time t.

Ehrenfest's Reply



Synge's 'snapshots' $\Longrightarrow$ hyperplane sections $t =$ const. of spacetime

> “ Question I: Is the last assertion accurate? If not, then what distinguishes the result obtained by the observer at rest through 'synchronous measurement' of the rotating disk from the result obtained by measuring the stationary tracing image Π_1?
>
> Question II: Assuming my assertion is valid, then the statements made by Mr. Ignatowski regarding the 'synchronously measured' circumference and radius are not entirely consistent [*widerspruchslos*]. They correspond to the following statements regarding the tracing images: The tracing image Π_1 has the same radius as Π, but its circumference is shorter. How can we imagine tracing images with such properties without any contradiction [*widerspruchslos*]? ”

(Ehrenfest 1910, 1129)

Ehrenfest's Reply

- The **two images**, Π and Π_1, show the same radius but different circumferences.
- This is unquestionably a **contradiction**, contrary to Ignatowski's claim.

⇓

> "It would be highly desirable, in case of further discussion, to avoid using the terms true and 'apparent' shape [*Gestalt*] of the rotating disk altogether, or, if this is not possible, to define the meaning of these terms with a simple and strict agreement."
>
> *(Ehrenfest 1910, 1129)*

Part VI

The Einstein-Varićak Correspondence

Ich habe deshalb im vorhergehenden versucht, von diesem Standpunkte aus einen Aufschluß über die Deformation bei bewegten Körpern zu erlangen.

Berlin, den 6. Februar 1911.

(Eingegangen 7. Februar 1911.)

Zum Ehrenfestschen Paradoxon.

Von V. Varićak.

Das Zustandekommen des Ehrenfestschen Paradoxons ist einleuchtend, wenn man sich auf den Standpunkt stellt, den Lorentz bei der Aufstellung seiner Kontraktionshypothese eingenommen hat, d. h. wenn man die Kontraktion des bewegten starren Körpers in der Bewegungsrichtung als eine objektiv stattfindende Veränderung ansieht. Unabhängig von dem Beobachter wird sich jedes Element der Peripherie nach Lorentz tatsächlich verkürzen, während die Elemente eines Radius unverkürzt bleiben.

Stellt man sich hingegen auf den Einsteinschen Standpunkt, demzufolge die besagte Kontraktion nur eine scheinbare, subjektive Erscheinung ist, verursacht durch die Art unserer Uhrenregulierung und Längenmessung, so erscheint mir jener Widerspruch nicht begründet zu sein.

Daß Herr Ehrenfest bei seiner Argumentation den Lorentzschen Standpunkt einnimmt, schließe ich aus den Fragen, die er an Herrn v. Ignatowsky gerichtet hat[1]), und hauptsächlich daraus, daß er jenen Widerspruch auch bei den Pausbildern Π und Π_1 zu finden vermeint. Mir will es scheinen, daß jene Pausbilder identisch sein müssen; sie werden denselben Radius und dieselbe Peripherie haben.

Um das zu begründen, sei es mir gestattet, auf die gleichförmige Translation eines starren Körpers zurückzugreifen, an dem gewöhnlich jene Kontraktion als Begleiterscheinung der Translation demonstriert wird. An dem vorderen Ende B des Stabes sei ein Spiegel befestigt und am rückwärtigen Ende A befinde sich eine Lichtquelle. Die doppelte Länge des Stabes messen wir durch die Zeit, welche ein Lichtsignal braucht, um von A nach B und zurück nach A zu gelangen. Um bei diesen bekannten Sachen nicht weitläufig zu werden, verweise ich z. B. auf die Arbeit von Lewis und Tolman[2]), die den radikalen Unterschied in den Lorentzschen und Einsteinschen Ansichten besonders stark betont haben. Dort kann man nachsehen, durch welche Überlegungen der ruhende Beobachter zur Annahme der Kontraktion des bewegten Stabes gezwungen wird. Aber er bleibt sich bewußt, daß diese Kontraktion sozusagen nur eine psychologische, und nicht eine physikalische Tatsache ist, d. h. daß der Körper in Wirklichkeit keine Änderung erfahren hat.

Nun führe der ruhende Beobachter mit diesem Stabe dasselbe Experiment aus, welches ihn Herr Ehrenfest mit der rotierenden Scheibe ausführen läßt[1]). Auf beiden Enden des Stabes sollen sich Marken befinden. Während der Stab ruht, hält der ruhende Beobachter ein Pauspapier P über ihm und paust die Marken auf das ruhende Blatt durch.

Während sich der Stab gleichförmig in gerader Linie vorwärts bewegt, hält der ruhende Beobachter ein Pauspapier P_1 über ihm und paust in dem Moment, wo seine Uhr auf t zeigt, mit einem Schlag beide Marken auf das ruhende Blatt durch.

Schließlich mißt der ruhende Beobachter B die Entfernung jener Marken an den ruhenden Pausbildern Π und Π_1 aus.

Ich glaube, daß er beidemal dieselbe Entfernung finden wird, denn der Stab ist in Wirklichkeit nicht kürzer geworden.

Das erwähnte Vorgehen des ruhenden Beobachters ist wohl identisch mit dem mechanischen Anlegen des Maßstabes an das auszumessende Objekt; das ist aber nicht dieselbe Operation wie das Ausmessen der Länge mit Hilfe der optischen Signale.

Kurz möchte ich noch erwähnen, daß bekanntlich die Uhren in den Punkten A und B des bewegten Stabes, obwohl sie gleichlaufend sind, verschiedene Zeiten angeben, wenn die Uhr des ruhenden Beobachters auf t zeigt.

Nur noch eine historische Bemerkung. Nachdem Lorentz die Hypothese aufgestellt hatte, daß alle Körper in Richtung der Erdbewegung eine Verkürzung ihrer Dimensionen erleiden, lag die Frage nahe, ob diese Deformation, bzw. Komprimierung, bei durchsichtigen Körpern nicht von einer Doppelbrechung begleitet ist. Die bezüglichen Versuche von Rayleigh und Brace ergaben ein negatives Resultat.

Nach dem Relativitätsprinzip von Einstein wäre man überhaupt auf die Frage nicht gekommen.

1) Diese Zeitschr. 11, 1129, 1910.

2) G. N. Lewis and R. C. Tolman, The principle of relativity, and Non-Newtonian mechanics. Proceedings of the american academy of arts and sciences **44**, 711, 1909. Neulich wurde auch von J. Laub mit Nachdruck darauf hingewiesen, daß in der Theorie von Lorentz die Auffassung dieser Erscheinung eine wesentlich andere ist wie bei Einstein. Siehe Jahrb. d. Radioaktivität und Elektronik 7, 430, 1910.

1) Ich lasse dahingestellt die Frage, ob dieser Versuch vom relativistischen Standpunkt aus als möglich zu betrachten ist.

Varićak's Paper

Varićak: The impossibility of setting a Born rigid body into rotation is understandable if one adheres to

- **Lorentz contraction** as an "objective change" (Varićak 1911, 169).
- **Einstein contraction** is only "an apparent, subjective phenomenon" fostered by the way we regulate clocks and measure lengths (Varićak 1911, 169).

⇓

> The [Einstein] contraction is only a psychological and not a physical fact, i.e. the body has not really undergone any change.
>
> *(Varićak 1911, 169)*

Varićak's Paper

Two rods, one at rest and one moving at constant velocity with respect to K:

- Mark the rods on the tracing paper P, which is used to label the set of all events happening at the same time t for the rest system K,
- Transfer these markings onto the second piece of tracing paper P' to create the two images Π and Π_1.
- Reproduce the two images Π and Π_1 on a non-transparent paper at rest.

> "I believe that the [stationary observer] will find the same distance both times, for in reality the rod has not become shorter."

(Varićak 1911, 169)

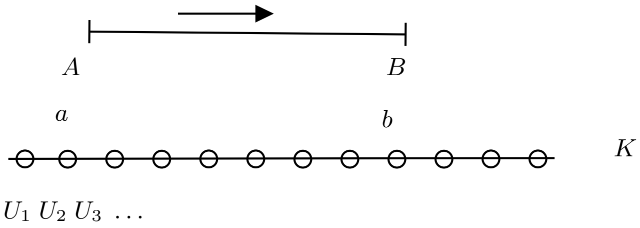
Einstein's Private Reply

Einstein received the draft around the same time and reacted in a friendly way, but critically:

> "Thank you very much for kindly sending the proof sheet, which I studied immediately. However, I completely disagree with its content and am quite certain that you are wrong. One must be very careful not to operate with the deceptive characteristics of 'real' and 'apparent'."

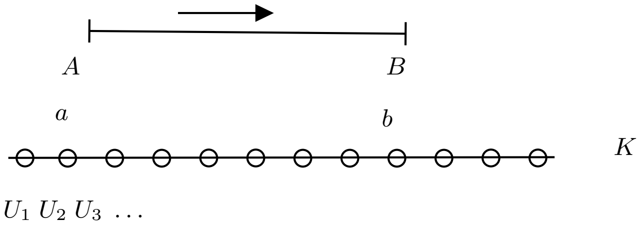
(Einstein to Varićak, Feb. 24, 1911; CPAE, Vol. 5[10], Doc. 255a)

Einstein's Private Reply



- K is a non-accelerated system along whose x-axis a series of clocks $U_1, U_2, U_3, \ldots$ are placed and synchronized using light signals.
- A bar AB moves with uniform velocity v along the x-axis of K.

Einstein's Private Reply



By the **length** of AB, we understand a number coordinated to it. There are two ways to obtain that number:

1: A measuring unit rod is accelerated without changing its length until it attains the velocity v, i.e., until it is at relative rest with respect to the bar AB:

- The length of AB is measured by successively applying the unit rod along the bar.
- The number l of measuring rods that can be aligned in this way can be called the "'real' length" of AB.

2: a group of synchronized clocks $U_1, U_2, U_3, \ldots$ are distributed along K

- one marks two points a and b on K where we can find the two ends of

Einstein's Private Reply

The results of both operations, 1 and 2, can equally be called **the length of the bar** AB Einstein: there is no *a priori* reason for operations 1 and 2 to lead to the same numerical value.

- classical kinematics predicts that $ab = AB$; that is, $l = l'$.
- relativistic kinematics predicts that $ab < AB$; that is, $l' < l$

Einstein's Private Reply

In principle, these predictions can be experimentally confirmed or disconfirmed:

- measure the moving bar AB using a comoving unit rod
- decelerate the latter while preserving its length
- measure the distance ab with the same unit rod

⇓

> "The contraction can be ascertained by measurement, i.e., it is 'real'"
>
> *(Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a)*

Einstein's Private Reply

Due to the relativity of the "definition of simultaneity", identical rods are used to measure the spatial distance between different pairs of points along the x-axis of different frames.

- Ignatowski and Varićak **relative = conventional**
- Einstein **relative $\neq$ conventional**

> "So that you can see that the contraction is not simply due to the definition of simultaneity in $[K]$, i.e., of a purely conventional nature, I add: it is impossible to adjust the clocks in such a way that even after the adjustment, the rod, when measured with the clocks, possesses the speed $\pm v$ and always has the same length l' with respect to $[K]$. From this, one can conclude with Ehrenfest that a rotation without elastic deformation is excluded according to the theory of relativity, if one adds that no transverse contraction occurs."

Einstein’s Private Reply

- the new **kinematics** does not allow for an increase in the angular velocity of the disk while keeping the rest length between neighboring points on the circumference constant.
- since the material of the disk resists deformation, the variation of angular acceleration must induce **stresses** within the disk’s material.
- Herglotz (1911) developed a relativistic theory of **elasticity** based on the assumption that stresses arise when the condition of Born rigidity is violated.

⇓

This is a result that is in principle **empirically testable**.

Einstein's Private Reply

Einstein contraction is just as '**real**' as the Lorentz contraction:

⇩

> One cannot ask whether the contraction should be understood as a consequence of the modification of molecular forces caused by motion or as a kinematic consequence arising from the foundations of the theory of relativity. Both points of view are justified. The latter corresponds roughly to Boltzmann's treatment of the dissociation of gases in terms of molecular theory, which is perfectly justified, although the dissociation laws can be derived from the second law without kinetics There is no (principal) difference with regard to the **result**, but only with regard to the **foundations** on which the investigation is based

(Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a)

Part VII

Real or Apparent: Einstein’s Public Reply to Varićak

Zum Ehrenfestschen Paradoxon.

Bemerkung zu V. Varičaks Aufsatz.

Von A. Einstein.

Neulich hat in dieser Zeitschrift[1]) V. Varičak Bemerkungen publiziert, die nicht unerwidert bleiben dürfen, weil sie Verwirrung stiften können.

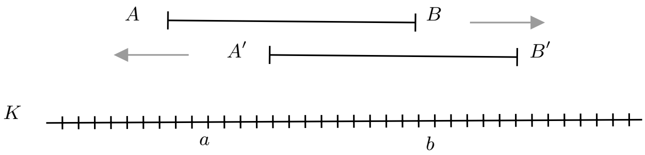
Der Verfasser hat mit Unrecht einen Unterschied der Lorentzschen Auffassung von der meinigen mit Bezug auf die physikalischen Tatsachen statuiert. Die Frage, ob die Lorentz-Verkürzung wirklich besteht oder nicht, ist irreführend. Sie besteht nämlich nicht „wirklich", insofern sie für einen mitbewegten Beob-

1) Diese Zeitschr. **12**, 169, 1911.

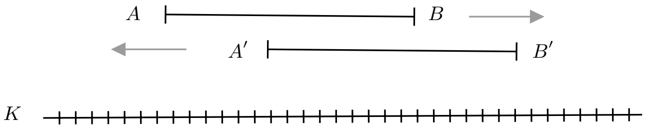
“Recently V[ladimir] Varićak published in this journal some comments that should not go unanswered because they may cause confusion. The author unjustifiably perceived a difference between Lorentz’s conception and mine *with regard to the physical facts*. The question whether the Lorentz contraction is real is misleading. It is not ‘real’ inasmuch as it does not exist for a moving observer; but it is ‘real’ i.e., in such a way that, in principle, it could be detected by physical means, for a non-comoving observer. This is just what Ehrenfest made clear in such an elegant way [. . .] But perhaps he might cling to the view that the Lorentz contraction has its roots solely in the arbitrary stipulations about the ‘manner of our clock regulation and length measurement.’ The following thought experiment shows the extent to which this view cannot be maintained”

(Einstein 1911b, 509)

Einstein's Public Reply to Varićak



- **two rods** AB and $A'B'$: when compared at rest in a non-accelerated coordinate system K, they have the same length $AB = A'B'$ sliding alongside each other with constant velocities v and $-v_1$
- one can imagine a **device that leaves a mark** a on the x-axis of K where the left end-points A and A' meet, and a mark b where B and B' meet, without the need to keep track of time.



The Twin-Rod Thought Experiment

- the theory of relativity **predicts** that the distance between the two marks, a and b, will be shorter than the length $AB = A'B' > ab$, whereas the old theory predicts that $AB = A'B' = ab$.
- this prediction can be **tested** empirically. By gently decelerating one of the rods—say, AB—so that it does not change its length , and laying it parallel to the x-axis along the two marks a and b in K.

Part VIII

Conclusion

Conclusion

- by the spring of 1911, Einstein has abandoned his theory of radiation, **relativistic theory of gravitation**
- **Ehrenfest visited Einstein** in Prague at the end of February 1912.
- Einstein made the first published reference to **Ehrenfest's thought experiment** in a paper on gravitation published in February 1912

⇓

the geometry of the rotating disk is
non-Euclidean (Einstein 1912, 356).

Conclusion

> “I ask you please to make allowances for my statements contained in *Kultur der Gegenwart*. Although I had 3 years of time to compose it, I had completely forgotten and was reminded of my commitment by Warburg one week before the delivery deadline. In this time I hastily pieced together the two articles as best I could. So please: do not punctiliously weigh every word! Regarding the erroneous view that the Lorentz contraction was ‘merely apparent,’ [*scheinbar*] I am not free from guilt, without ever having myself lapsed into that error. It is real [*wirklich*], i.e., measurable with rods and clocks, and at the same time apparent [*scheinbar*] to the extent that it is not present for the co-moving observers”

(Einstein to Lorentz, Jan. 23, 1915; CPAE, Vol. 8, Doc. 47)

Conclusion

- **length contraction** is apparent because it disappears for the comoving observer, but is real since it can be tested empirically $\rightarrow$indirectly: the appearance of Ehrenfest's stresses
- **time dilation** is apparent because it disappears for the co-moving observer, but it is real since it can be tested empirically $\rightarrow$directly: transverse doppler effect

$$\Downarrow$$

- **Varićak and Ignatowski** took for granted the conceptual opposition between 'dynamical = real' and 'kinematical = apparent'
  - so did Bell and the physicists at CERN $\Longrightarrow$ Lorentzian pedagogy
- **Ehrenfest and Einstein** redefined the terms as 'experimentally testable = real' vs. 'frame-dependent = apparent'.
  - so do most standard textbooks Moeller, Rindler, etc.. $\Longrightarrow$ Einsteinian pedagogy

Today Philosophical Debate

Brown (2005) and others: Bell paradox $\Longrightarrow$ **dynamical** interpretation of SR

- **theoretical claim**: we should adopt Lorentzian philosophy
- **historical claim**: Einstein shared the Lorentzian philosophy

This paper: Ehrenfest paradox $\Longrightarrow$ **kinematical** interpretation of SR

- **theoretical claim**: is a non-sequitur
- **historical claim**: is false

Real and Apparent in General Relativity: Einstein's 1918 'Dialogue'

> First, I must point out that the distinction between real and non-real can be of little help to us. With respect to K', the gravitational field exists in the same sense as any other physical object that can only be defined with reference to a coordinate system, even though it is not present with respect to the system K. [...] Instead of distinguishing between 'real' and 'non-real', we should more clearly differentiate between quantities that belong to the physical system itself (independent of the choice of coordinate system) and those quantities that depend on the coordinate system.

Einstein, 1918

Thanks!

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