# The Philosophical Coming of Age of Science.
## Euler's Role in Cassirer's Early Philosophy of Space and Time

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Expanded Version

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Cassirer's early philosophy of space and time, overshadowed by his later work on relativity, has been scarcely explored in the literature. This paper aims to bridge this gap. It argues that understanding Cassirer's point of view requires acknowledging the pivotal role he attributed to the work of Leonhard Euler in the philosophical 'coming of age' of modern science. Against the Leibniz-Berkeley *philosophical* plea for the relativity of all motion, Euler objected that if Newton's absolute space and time did not exist, the principle of inertia would be come meaningless and with it a *scientific* theory of motion. According to Cassirer, Kant took a step beyond Euler by shifting the focus from the *existence* of space and time as 'things' to their *function* as necessary 'conditions' of the possibility of mechanics. In the nineteenth century, it became clear that Newton's absolute space and time entail more structure than necessary. Nevertheless, according to Cassirer, the Euler-Kant insight still holds: a geometric structure serving as an inertial structure is the *condicio sine qua non* of a coherent theory of motion, including general relativity. This paper concludes that Cassirer came close to defending a sort of 'inertial functionalism' dressed in neo-Kantian garb.

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*Keywords:* Ernst Cassirer • Marburg neo-Kantianism • Space and Time • Leonhard Euler • Inertial Frame

## Introduction

Cassirer's philosophy of space and time has usually been discussed through the lens of his writings on Einstein's relativity theory published in the 1920s (Cassirer 1921; see Ryckman 2005, Chap. 2). However, to my knowledge, it has not been noted that Cassirer's interest in this topic goes back at least two decades earlier (Cassirer to Natorp, Nov. 26, 1901; ECN, Vol. 18, Doc. 43). This oversight likely stems from the fact that Cassirer's early remarks on the topic are somewhat 'buried' beneath the sheer mass of his extensive historical (Cassirer 1906a, 1907) and theoretical work (Cassirer 1910) from the late 1900s, which had a much broader scope. However, while Cassirer's early contributions to the history and philosophy of physical spacetime theories might go unnoticed on cursory reading, more careful scrutiny unveils an original and profound insight that not only sheds new light on his subsequent related work, but, to some extent, is more compelling than the latter.

This paper contends that to appreciate Cassirer's viewpoint, it is useful to dwell on the pivotal role he assigned to the work of Leonhard Euler in a lengthy section of *Das Erkenntnisproblem* (Cassirer 1907, Chap. 7.2.II.2b). Euler's *scientific* contributions to the history of mechanics were well known during Cassirer's time, although they were often treated as mere 'footnotes' to Newton's work (Mach 1883; see Truesdell 1968, Chap. 2). Cassirer deserves recognition for having emphasized, I believe for the first time, the *philosophical* impact of Euler's work on eighteenth-century thought. In particular, Cassirer interpreted Euler's reflections on space and time as the philosophical 'coming of age' (*philosophische Mündigkeitserklärung*) of the new mathematical science of nature (Cassirer 1907, 477). By defending Newton's concept of absolute space and time against the 'relativistic' objections of Leibniz and Berkeley, Euler established science over philosophy as the touchstone of objective 'knowledge'. Philosophy does not have authority over the concepts created by mathematical physics; instead, it must recognize these concepts as the fundamental 'fact' that serves as the starting point for its own investigations (Cassirer 1919, 26f.). Thus, Cassirer goes so far as to present Euler's stance toward philosophy as the embryonic form of what would later evolve into Kant's 'transcendental method'—that is, for a neo-Kantian philosopher, the 'method of philosophy' *tout court*: the analysis of scientific knowledge as an objective historical 'fact' to search for the conditions of its possibility (Cohen 1885, 66–79).

To counter Berkeley's and Leibniz's philosophical defense of the principle of relativity of all motions, Euler argued that the *existence* of Newton's absolute space and absolute time is proved by the fact that, without them, the principle of inertia and consequently the entire system of scientific mechanics would become meaningless. In Cassirer's reading, Kant's philosophy of space and time can be regarded as an attempt to translate Euler's *ontological* concern into a purely *methodological* one. If Euler asked whether space and time *exist* in themselves, Kant shifted the question to the *function*[1] that they play within a viable theory of motion. As Cassirer put it with one of his favorite puns, Kant transformed space and time from 'things' (*Dingen*) that exist into conditions (*Bedingungen*) necessary for the possibility of Newtonian mechanics. It is because they play this *function* that they should be regarded as *a priori*. Cassirer showed that in the nineteenth century, especially with the work of Ludwig Lange (Cassirer 1910, Chap. 10.VI), it became clear that Newton's absolute space and time entailed more structure than was strictly required by Newton's theory. However, according to Cassirer, the Euler-Kant insight that a certain objective geometrical structure is required for the formulation of a coherent theory of motion could not be questioned even by the theory of relativity.

Cassirer was not an 'analytic philosopher'—a technical 'philosopher of physics' in modern terms. Such a professional figure, paradigmatically embodied by Hans Reichenbach, was only starting to emerge around that time. Cassirer, like most neo-Kantian philosophers of that generation, was a 'synthetic philosopher',[2] whose reflections on physics were ultimately supposed to be part of a larger project aimed at investigating other cultural forms, such as religion, myth, art, etc. (Cassirer 1923–29). However,

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[1] Cassirer's use of the term 'function' is somewhat ambiguous. (a) In most cases, he uses 'function' in a mathematical sense, as when he famously contrasts 'function-concepts' with 'substance-concepts' (Cassirer 1910). (b) However, on several occasions, he uses 'function' to refer to the 'role' that a particular concept plays within a specific theoretical structure. On this point, see Heis 2014, 253. In the following, I refer exclusively to meaning (b).

[2] For this distinction, see Schliesser 2019.

with the wisdom of hindsight (after Stein 1967), Cassirer's historical-critical analysis of the evolution of the scientific theory of motion seems to be more 'modern' than Reichenbach's logical-mathematical analysis.[3] In particular, contrary to Reichenbach (1924), Cassirer sees quite lucidly that Einstein was the heir of Newton and Euler rather than Leibniz and Berkeley.

Cassirer seems to have realized that no theory of motion, including general relativity, is possible without an 'inertial structure' that provides a standard of non-acceleration. For Cassirer, the choice of a spatiotemporal structure sufficient for this purpose is neither empirical nor conventional; it is ultimately determined by the *function* it plays within the mathematical theory of motion as a whole. In this sense, this paper concludes that Cassirer appears to have come close to defending a sort of 'inertial functionalism' (Baker 2020) dressed in neo-Kantian garb.[4] Following his student at Yale, Arthur Pap (1943), one might attribute to Cassirer a 'functional-pragmatic interpretation' of the *a priori*[5]. The *a priori* is necessary in the hypothetical sense of the expression: as the history of spacetime theories shows, an 'inertial structure' is necessary *if* a coherent dynamic theory of motion is to be possible. *That* this is the case is a contingent 'historical fact' in which philosophy has no say.

## 1 Cassirer on the Reception of Newton's Work in the Eighteenth Century

In the 'Preface' to his first 1902 book on Leibniz, Cassirer announced that he was working on a more ambitious project on the 'prehistory of criticism' in the mathematical science of nature of the eighteenth century (IX). The aim was to present Kant's philosophy as the outcome of an historical process shaped by figures like Galileo, Leibniz, Descartes, and Newton (Ferrari 2015). Cassirer appeared to have already concluded the parts on space and time that he initially planned to publish in a journal (Cassirer to Natorp, Nov. 26, 1901; see also Cassirer 1904, 1:108–111). However, the project quickly expanded beyond his initial expectations. It was only by 1905 that Cassirer could announce the completion of the first volume of *Das Erkenntnisproblem* (Cassirer 1906a), which covered the period from the Renaissance to Descartes (Cassirer to Natorp, Jul. 31, 1905; ECN, Vol. 18, Doc. 70). A second volume, covering the period from Bacon to Kant, was already in preparation and was published in 1907. By that time, Cassirer had obtained the *venia legendi* at the University of Berlin, with a *Probevorlesung*, "Substanzbegriff und Funktionsbegriff," which outlined the systematic framework of his historical scholarship (see Ferrari 1988).

Cassirer's *Das Erkenntnisproblem* aimed to illustrate the intricate and sometimes conflicting interplay between the history of the 'philosophical problem of knowledge' and the history of the 'mathematical science of nature'. In Cassirer's view, this dynamic initially appeared "to have reached a secure conclusion in Newtonian science" (Cassirer 1907, 401). In fact, for his followers, Newton was not primarily the discoverer of the 'law of gravitation', but above all the founder of a new method of research, the 'method of induction': "His work represents for them a philosophical act, insofar as in it the inductive method not only achieved its highest results, but also reached its first

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[3] Cassirer attributes to Euler a stance that modern commentators tend to ascribe directly to Newton (see DiSalle 2006).

[4] Baker (2020) refers to Knox's (2013) space-time functionalism, according to which what counts as 'spatiotemporal structure' is determined by its *function* in defining the 'inertial structure' presupposed by the laws of motion. See Cassirer's use of the term 'function' in fn. 1.

[5] See Stump 2020.

logical articulation and establishment” (Cassirer 1907, 401). The Newtonians elevated ‘induction’ to the sole source of physical certainty, advocating for a ‘physics without hypotheses’.

To be sure, Cassirer pointed out, Newton aimed not to abolish metaphysics but to carefully demarcate it from the mathematical science of nature. However, “[o]nly at one point does Newton depart from this critical restraint: His scientific doctrine of space and time” (442). Newton conceded that, from the perspective of pure observation, all knowledge of spatial and temporal determinations dissolves entirely into *relations*. The question of the existence of space and time in themselves, beyond these observable relationships of bodies, seems forbidden. And yet, Cassirer continued, Newton’s *Scholium* to the Definitions of space, time, and motion in the *Principia* famously placed precisely these concepts at the forefront and continuously asserted their necessity for the foundation of mathematical physics (463f.). How does Newton justify his claim that freely moving bodies travel in straight lines relative to absolute space, even though he concedes if absolute space is unobservable? How is Newton certain that the oscillations of a perfect pendulum occur at consistent intervals in relation to absolute time, if the flow of time is unobservable? If our experiences are confined to relative spaces and times, and if experience sets the limits of our knowledge, then we should not be allowed to postulate the existence of absolute space and time (463f.).

It seems that an obscure metaphysical concept, borrowed from Henry More’s speculative Platonism (442–446),[6] is embedded in the foundations of mathematical physics itself: “But with that, the power of pure induction, as Newton understood and proclaimed it, would already be broken” (464). Among the methodological rules Newton presents for research, the first demands that, in explaining phenomena, one is not allowed to resort to any other than ‘true causes’ that can be directly observed. “The existence of absolute space and absolute time, however, is not a ‘*vera causa*’ in this sense” (465). It is, as Newton himself emphasizes, not given directly by experience, since all our empirical observations are restricted solely to relative spaces and times. However, according to Newton, not only does mechanics require the concept of absolute motion, but it also enables us, in empirical cases, to provide criteria to recognize it and indirectly demonstrate its effects. As is well known, in the case of rotational motion, the appearance of centrifugal forces seems to be completely independent of whether the body rotates with respect to bodies immediately surrounding it. Therefore, centrifugal forces provide us with a reliable indicator that allows us to distinguish between ‘apparent’ rotation with respect to other observable bodies and ‘true’ rotation with respect to an unobservable absolute space (Cassirer 1904, 1:110).

Ultimately, Newton could not avoid postulating the existence of an unobservable entity to explain observable phenomena. According to Cassirer, “[i]n this conflict lies the internal crisis of the Newtonian theory of experience” (Cassirer 1907, 465). ‘Physicists’ began to question this assumption, albeit cautiously at first due to the overwhelming authority of Newton. However, ‘philosophers’ such as Berkeley and Leibniz engaged in a frontal attack. Although starting from very different premises, both contested Newton’s doctrine of the existence of absolute space and time:

*Leibniz.* Against Newton’s reification and metaphysical hypostatization of space, Leibniz famously emphasized that space and time are “ideal orders of appearances”

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[6] Cassirer’s source is possibly Lange 1886a, §4 and 47f.

(Cassirer 1907, 187). Due to the homogeneity of Euclidean space, absolute differences in absolute place are not real differences (Cassirer 1902, 246–250). If a body is displaced, there is no way to distinguish between the original and the displaced situations without a reference body that does not participate in the displacement. Indeed, if we were to move the entire universe in an otherwise empty space—as Leibniz's celebrated argument in the correspondence with Clarke goes[7]—this motion would change nothing. It cannot, in principle, be observed. Since only the relative positions of bodies have empirical significance, Leibniz concluded that, in principle, all states of motion are equivalent (equivalence of hypotheses; see Cassirer 1904, 1:109). If Clarke objected here that the fact of change is not conditioned by its observation, then Leibniz responded by introducing his *principe de l'observabilité*: When there is no *possible* observable change, there is no change at all (Cassirer 1902, 249).

*Berkeley.* If Leibniz raised 'logical' objections against Newton's absolute space, Berkeley (1721), according to Cassirer, sought to attack the latter using the tools of 'psychological' criticism. His critique of the doctrine of space is rooted "in polemics against abstract concepts" (Cassirer 1907, 465). We are not allowed to isolate a simple property observed in specific circumstances and treat it as an independent content (465f.). Even the supreme laws of mechanics, such as the principle of inertia, "cannot contain in themselves any moment that is not indirectly or directly rooted in experience and can be substantiated in it" (467). The affirmation that every body left to itself persists in its state of rest or of uniform rectilinear movement "loses none of its value if, instead of referring the continuation of the movement of the body to absolute space, we commensurate it with the position of the body with respect to the sky of the fixed stars" (467).

Leibniz and Berkeley seem then to achieve the same *ontological* conclusion: "The rejection of the absolute reality of space and time" (467). However, they started from opposite *methodological* premises: "The 'abstraction,' which for Berkeley is the source of error, represents for Leibniz the foundation of all rational and scientific understanding" (469). Berkeley's sensualist theory excludes absolute space and absolute time because both are not given in *actual* observation, the sole warrant of reality; for Leibniz's rationalist approach, given the perfect uniformity of mathematical space and time, differences in absolute positions in an empty uniform space cannot be the object of *possible* observation.

According to Cassirer, the objections raised by Leibniz and Berkeley against Newton's doctrine of space and time, precisely because of the disparity between their philosophical premises, defined the boundaries of the 18th-century debate (470). Physicists, even those of Leibnizian persuasion, ultimately sided with Newton; philosophers of all schools tended to agree with his opponents (470). In Cassirer's view, the struggle between Leibniz and Berkeley on the one hand, and Newton on the other reveals more than a disagreement on technical matters; it was the manifestation of a deeper tension between philosophy and science. For the philosophers, the principle of relativity of all motions was the inevitable conclusion of both the logical and psychological analysis (Cassirer 1904, 1:110). However, the entire historical development of mechanics seemed to directly contradict such a premise. It turned out to be impossible to build a coherent theory of motion while maintaining a relativistic view (1:110). The resolution of this

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[7] Cassirer just edited the German translation, Leibniz 1904, 120-241.

divergence, Cassirer argues, could only occur when the real crux of the dispute was clearly identified: “It is Leonhard Euler who accomplishes this feat, thereby ushering the general problem into a new phase” (Cassirer 1907, 472).

Cassirer notes that, in Euler’s view, in disputing the Newtonian concepts of space and time, Berkeley seems to agree with Leibniz on all counts; “this apparent unity extends so far”, that he “in his defense of the ‘mathematician’ Newton against the ‘philosophers,’ juxtaposes the two philosophical lines of thought indiscriminately” (Cassirer 1902, 261). According to Cassirer, this was not simply the consequence of Euler’s alleged philosophical naivety. On the contrary, Euler lucidly understood that the philosophical quarrel between ‘rationalism’ and ‘empiricism’ did not grasp the issue at stake (Cassirer 1904, 1:112). Indeed, representatives of both schools, although starting from different premises, pursued the same strategy. They evaluated Newton’s ‘mathematical’ theory of space and time against a purely ‘philosophical’ standard (*Maßstab*). However, according to Euler, the proper standard cannot be taken either from pure reason or from pure experience. It can only be derived “from the pure dynamic laws; they are the content with which space and time must be filled in order to become natural realities” (Cassirer 1902, 461).

## 2 Cassirer on Euler’s Defense of Newton’s Absolute Space and Time

According to Cassirer, Euler’s first comprehensive work on mechanics, published in 1736, already posited the decisive problem without being able to offer a solution. ‘Motion’, as it appears in the initial naive observations, simply presents itself as a process of change in the ‘relative place’—a finite portion of space with respect to which we decide on the state of motion or rest of bodies. However, ‘place’ as such can only be determined as ‘absolute place’—a part of the boundless or infinite space in which the material world is contained (Vol. 1, Chap. 1, Sect. 8, p. 14.). However, Cassirer noted that Euler realized that the logical and ontological *status* of this boundless and infinite space was hard to grasp. On the one hand, Euler asserts that pure mathematical space is nothing more than an ‘idea’; it does not concern us whether it really exists (Cassirer 1907, 407). On the other hand, it becomes clear that we cannot fully grasp this ‘idea’ with everything that the senses and imagination present to us (Euler 1736, Vol. 1, Chap. 1, Sect. 77, p. 32). According to Cassirer, Euler seems to be at his wits’ end: “If we consider absolute space in light of the ordinary metaphysical fundamental distinction between physical or psychic being, we immediately see it placed in an untenable middle position: The sphere of the ‘subject’ as well as that of the ‘object’ seems to exclude it in equal measure” (Cassirer 1907, 474).

Euler addressed the issue head-on only over a decade later in his ‘Réflexions sur l’espace et le tems’, presented to the Berlin Academy in 1748 (Euler 1750). According to Cassirer, for the first time, Euler makes it clear that neither logical-psychological nor ontological-metaphysical considerations are adequate to solve this problem: “The touchstone [. . .] can lie nowhere else than in the principles of *scientific mechanics* and in the *laws of motion* that mechanics places at the forefront. These laws are so firmly established and of such incontrovertible certainty that they must serve as the sole foundation for all our judgments about the world of bodies: And they maintain this value, regardless of whether it is possible to derive them from allegedly higher principles of metaphysics or not” (Cassirer 1907, 475). Regarding the nature of space and time, we can only gain a firm guideline when we consider these concepts not in isolation but rather in the "relational context [*Verhältnisstellung*] and connection they enter into with each other in the principle of inertia [*Prinzip der Beharrung*]" (Cassirer 1907, 475), whose truth is removed from all philosophical disputes of the schools.

The various factions, both rationalists and empiricists, did not contest the content of the principle of inertia; instead, they sought to explain it through their respective *philosophical* concepts of space, time and motion (476). Cassirer suggests that for Euler this was the crux of the issue: our philosophical concepts should be evaluated against the principle of inertia, the only legitimate *scientific* standard: "The decisive question is not what space and time *are* [*sind*] in and of themselves, but rather what they are *used* [*gebraucht*] for in the enunciation and formulation of the law of inertia" (476). If the consideration of relative positions and relative motions suffices to make the content of the principle of inertia understandable, then we could forgo the notions of absolute space and time; however, if it turns out conversely that the principle of inertia is meaningful only if we admit an absolute space and an absolute time, then the necessity of these concepts is established (476). The objection that we are illegitimately hypostasizing our own 'ideas' becomes moot. Indeed, Euler (1750, 382) could simply declare as absurd the notion that a pure fiction of our imagination could serve as the basis for the principles of mechanics and account for their success (Cassirer 1907, 476). According to Cassirer, in this manner, Euler introduced "a new path" (476) between rationalism and empiricism, putting the entire debate on the nature of space and time on a new footing:

> The true nature of space and time is not revealed to us by immediate sensory observation—but also the psychological analysis of ideas cannot lead us to the goal. The essence of both is rather to be determined solely by the *function* they fulfill in the system of mathematical physics. The set of the mechanical principles, since it constitutes the prerequisite for all exact explanations of phenomena, forms the *Archimedean* point of our knowledge. Vague speculative endeavors are met here with a solid fact that cannot be pushed aside or interpreted away. Euler's doctrine is the philosophical declaration of maturity [*Mündigkeitserklärung*] of the new mathematical science, which henceforth undertakes to establish from within itself the true standard of 'objectivity,' instead of allowing it to be imposed by any foreign interest. Philosophy, as now sharply and unequivocally stated, should not master experience, but merely understand it and lay bare its foundations. (476f.)

The principle of inertia states that a body, as long as no external forces act upon it, remains in a state of rest or uniform rectilinear motion. But rest or uniform motion—relative to what? Perhaps to the fixed stars, Euler suggests, possibly alluding to Berkeley (479). However, we cannot exclude the possibility, or even the likelihood, that the fixed stars are only apparently at rest with respect to one another. Ultimately, the principle of inertia is not strictly valid for any empirically given reference body; yet this law must have absolute truth, because without it the entire science of dynamics would collapse, including Newton's laws of motion and the law of gravitation. Anyone who acknowledges these laws must also acknowledge the reference systems for which they are valid; and no empirically given body suffices, but only the 'absolute space' (479).

According to Cassirer, Euler's line of argument implicitly reshaped the relations between philosophy and the exact sciences. If our 'philosophical' concepts prove insufficient to grasp the content provided by physical science, then the inadequacy lies solely with those concepts. Therefore, we must refine them until they become fully compatible with the secure content of physical science: "In this view, Euler only fixes the general ideal that had constantly hovered over the exact research of the time" (Cassirer 1907, 477). By upholding this general criterion, the objections of Leibniz and Berkeley can be seen in a new light:

- The *rationalists* who, in the name of principle of relativity of all motions, deny the concepts of absolute space and absolute time, must simply try to use their own definitions to see to what extent they are able to construct with them a coherent theory of motion compatible with the principle of inertia: "However, one only needs to attempt this experiment once to be immediately convinced of its impracticability" (478). A fully 'relativistic' theory of motion lacks the notion of constancy of direction over time without which principle of inertia is meaningless.
- A reference system must be tacitly presupposed when we attribute uniform velocity and direction to a body moving on its own. However, no reference system can meet the *empiricist*'s demand, as no observed body is entirely at rest (479). Any attempt to present the principle of inertia as an empirical proposition describing the relations among individual bodies, like moving bodies and fixed stars, would threaten the status of the principle of inertia as a universal principle: "Indeed, we are not dealing with the establishment of a single fact in it, but with an ideal norm by which we judge all natural phenomena" (479)

Thus, according to Cassirer, Euler always returns to the same point: the very possibility of mechanics requires that we postulate the existence of absolute space and absolute time: "Both concepts possess undeniable reality" not because they are validated solely by pure experience or pure reason, "but because—a fact that carries greater importance—they are indispensable to the entirety of our scientific understanding of the world" (479). The principle of inertia, as a universal principle, is meaningful only if one postulates the existence of absolute space and time.

According to Euler, 'philosophers'—whether empiricists like Berkeley or rationalists, like the representatives of the Leibniz-Wolffian School—have misunderstood this point. They treat the concepts of space and time as "mere abstracts, thereby denying them their true concrete [*gegenständlichen*] content" (480). However, their conclusion is ultimately rooted in the ambiguous nature of the very notion of 'abstraction'. Of course, an intellectual effort is required to elevate oneself to the idea of pure mathematical space and time. Yet, the method of 'reflection' involved in this endeavor is fundamentally different from how we typically form general concepts of genus and species. (480). The concept of 'place' cannot be obtained in the same way as, for example, the concept of 'yellow' as the property common to lemons, daffodils, canaries, etc. In fact, the 'place' where a thing is located is not a 'property' that belongs to the thing alongside its other characteristics (480). In contrast, the idea of 'place' results when one thinks of the body as a whole as removed, so that place cannot have been a 'property' of the body (480).[8]

However, as Cassirer pointed out, Euler now faces a fundamental philosophical difficulty. If one wishes to obtain the correct 'psychological' correlate for the space of mathematical physics, then we must "insert a new intermediate link between perception and concept, between concrete sensation and abstract thought, as interpreted by the school tradition in logic" (481). The conventional separation proves inadequate when compared to the concepts of exact science: the analysis of objective scientific knowledge urges a reorganization of our *philosophical* categories. Euler could not find the conceptual tools apt to grasp the nature *sui generis* of space and time. He could

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[8] See Euler 1750, §XV.

simply conclude that, since pure space and pure time are necessary for the formulation of the laws of motion, they must exist as separate things: “It is Euler’s self-evident and unquestioned assumption that for the objective significance of the principles, a substantive correlate in absolute being is to be assumed and demanded” (Cassirer 1907, 481).

Nevertheless, Cassirer continued, Euler himself seems to have been uncomfortable about the status of this presupposition. Indeed, according to Cassirer, Euler’s last comprehensive presentation of mechanics, the *Theoria motus* from the year 1765, shows that his ‘philosophical’ scruples could not be permanently appeased. After nearly two decades, Euler returned to the problem anew and presents both opposing views, the relativistic and absolutist, side by side without initially taking stance (Cassirer 1907, 484). In the first part of the book, motion’s relativity is shown as the inevitable conclusion when considering motion solely through direct sensory perception. However, in the second part of the book Euler insists again that this conclusion is clearly incompatible with “the highest principles of mechanics” (484). Euler’s presentation is not simply inconsistent, as some interpreters contended (Streintz 1883, 45). Rather, it places us in front of the fundamental paradox that what cannot be an object of experience, absolute space and time, is required for the construction of a theory of motion compatible with experience: “The ‘abstraction,’ which was just rejected, must therefore be reinstated and restored to its rights” (Cassirer 1907, 484).

In Cassirer’s reading of this conundrum, one observes “[t]he motive that dominates Euler’s entire intellectual development” (484). To make sense of scientific experience, he is continually compelled to move from *sensory* perception back to pure *conceptual* constructions like absolute space and absolute time that have no direct empirical counterpart. However, once he is forced to admit the *existence* of such unobservable entities, “the boundary between exact science and metaphysics seems once again abolished” (484). The concepts of absolute space and absolute time cannot be mere fictions of our imagination: They are implicitly contained in Newton’s laws of motion, whose success is undeniable. However, their particular manner of objective reality cannot be compared to that which we ascribe to physical bodies; indeed, we assume that they continue to subsist even if the latter are moved elsewhere or even completely removed. Euler’s analysis of notions of space and time deals a fatal blow to both traditional rationalist and empiricist theories of ‘concept formation’: “[t]he ‘categories’ [*Klassen*] of metaphysics had to be first broken in order to grant the new reality—as required by exact science and its laws—its rightful place within the overall system of knowledge” (485). Here, the destructive aspect of Euler’s argument is taken to its logical conclusion. However, according to Cassirer, Euler ultimately failed to articulate a constructive proposal, namely, to provide a new set of conceptual tools capable of expressing the peculiar status of space and time.

As the reader might already anticipate, in Cassirer’s reading, it was Kant who was able to provide the *pars construens*, or at least to indicate a way toward it. In his 1768 treatise on space, Kant explicitly aims to take one step further back along Euler’s line of argumentation (AA 2:378): Not only does the ‘fact of mechanics’, but already the ‘fact of geometry’ requires space to exist independently of relations among specific bodies (689).[9] Once again, however, one faces the paradox that space is something that is neither a body nor a property or relation of bodies; yet, it is difficult to determine in

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[9] Kant’s ‘incongruent counterparts’ are, so to speak, the geometrical equivalent of Euler’s principle of inertia.

what sense it 'exists' as an unobservable empty container. In Cassirer's interpretation, Kant identified the key to resolving the paradox during the critical period. The problem of the *ontological status* that space and time possess with respect to material bodies is transformed into that of the *methodological function* that they play within geometry and mechanics. Space and time are empirically real—not in the sense that they exist as independent objects—but because they are *a priori* conditions[10] of the possibility for all our objectively valid empirical knowledge of the disposition and motion of bodies: "To pure space and pure time belongs the objectivity of the condition [*Bedingung*], while the objectivity of the thing [*Ding*] remains precluded" (Cassirer 1907, 700f.).

## 3 Cassirer and the Problem of the 'Reference Frame'

Early on, Cassirer planned to integrate the first two volumes of *Das Erkenntnisproblem* with the third, a systematic volume (Cassirer to Natorp, Jul. 31, 1905; ECN, Vol. 18, Doc. 70), that ultimately became his monograph *Substanzbegriff und Funktionsbegriff* in 1910. The sections of *Das Erkenntnisproblem* that describe the historical evolution of the 18th-century debate about the nature of space and time from Newton to Euler (Cassirer 1907, Chap. 7.2) have their theoretical counterpart in the section of *Substanzbegriff und Funktionsbegriff* dedicated to the late 19th-century German-speaking debate on the meaning of the principle of inertia from Neumann to Lange (Cassirer 1910, chap 4.VI). Although many readers today might be aware of the latter debate (see DiSalle 1988), it is worth noting that, as far as I can see, Cassirer was the first and possibly the only philosopher to have attempted to provide an overview and a philosophical appreciation of it.

As Cassirer pointed out, in the 18th century, the controversy about the nature of space and time focused exclusively on the contrast between 'absolute' and 'relative' spatial and temporal determinations. However, no physicist, Cassirer argued, has ever taken that the concept of 'absolute' space would literally exclude consideration of any reference frame whatsoever: "The conflict is only with regard to the kind of system of reference, only with regard to whether it is to be taken as material or as immaterial, as empirically given or as an ideal construction" (Cassirer 1910, 229; tr. 172f.). However, it was only in the late nineteenth century that the 'problem of absolute space' explicitly became the 'problem of the reference-system': relative to what system of reference is the motion of a free body rectilinear; relative to what time-scale is it uniform?

### *3.1 Neumann and the Alpha Body*

Cassirer, as most historians do today, credited Carl Gottfried Neumann, son of the physicist Franz Neumann, for having firs formulated the problem in these terms in his 1869 academic inaugural lecture at Leipzig *Ueber die Principien der Galilei-Newton'schen Theorie* (Neumann 1870). The principle of inertia asserts that a material point, when set in motion and left entirely to itself without any external force acting upon it, moves in a straight line and covers equal distances in equal intervals of time. However, Neumann objected, the concept of motion in a straight line remained

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[10] Cassirer emphasized the Eulerian roots of Kant's Metaphysical Exposition of space (B37–40): in particular, the claim that space is not an abstract genus concept. However, space is also not a concrete individual thing that can be given in sensible intuition. In Cassirer's reading, Kant's notion of 'pure intuition' was meant to provide the philosophical 'class' that Euler was missing. See also, Natorp 1910, 273.

undefined. For instance, a trajectory that appears straight when observed from Earth may appear curved when viewed from the perspective of the Sun. In principle, any motion that appears straight relative to one celestial body will appear curved when observed from another, rendering the very notion of inertial motion devoid of physical meaning. The spatial part[11] of the principle of inertia, the fundamental principle of mechanics, and thus of the mathematical science of nature in general, as it is commonly presented, not only lacks any immediate *empirical* significance but is also *logically* incomprehensible—(*Begreifflich, Verständlich*) —without any clear meaning (Cassirer 1910; tr. ). It becomes 'logically contradictory' unless we specify the system to which we want to refer the freely moving point, especially if we attribute to it a certain 'straight-line' motion.

According to Cassirer, Neumann's solution to the problem was, indeed, a paradox, but not a gratuitous one. It served awaken us to the bizarre nature of the fundamental principle of Newtonian dynamics: "According to Neumann, the principle of Galileo can only be grasped in its conceptual meaning through the assumption of a definite existential background [*Daseins-Hintergrundes*]. Only in a world in which there exists at an unknown point of space an absolutely rigid body, unchanging in its form and dimensions to all time, are the propositions of our mechanics intelligible [*verständlich*]" (Cassirer 1910, 238; tr. 180). Since no observable body can be found that serve the purpose, Neumann introduced an unobservable one Neumann famously labeled this unknown body as 'Alpha Body'. The content of the principle of inertia becomes then that a material point left to itself moves in a straight line with respect to *one and the same* object, the Alpha Body. Only if the existence of such body is admitted, the spatial part of the law of inertia becomes 'logically understandable', that is non-contradictory.

According to Cassirer, one cannot help but be perplexed when encountering such arguments in physics. The *existence* of an unobservable single body, the Alpha Body is deduced by means of a purely logical inference. Indeed, in Cassirer's reading, Neumann's argument for the existence of the Alpha Body appears as a sort of physical version of the 'ontological argument' for the existence of God: an empirically unknowable existence is posited through pure thought, driven by the necessity of logical consistency of classical mechanics (Cassirer 1910, 239; tr. 180). And to this existence, although it is to be of material nature, are ascribed all those predicates usually employed by the ontological argument: it is characterized as immutable, eternal, and indestructible. Thus, on the one hand, the 'existence' of an entity is deduced solely from thought; on the other hand, the the logical possibility of an abstract concept is contingent upon the 'existence' of such entity: "If we conceive the Alpha Body annihilated by any force of nature, the propositions of mechanics would necessarily cease not only to be applicable, but even to be *intelligible* [*verständlich*]" (Cassirer 1910, 238f.; tr. 180).

For Cassirer, Neumann's cure is clearly worse than the disease. In Neumann's view if centrifugal effects reveal a body to be rotating but there is nothing observable relative to which it is rotating, then we must assume that it is rotating relative to something that we cannot see. All statements of classical mechanics hinge on something fundamentally unexperiencable; they describe the relationship of known bodies to a completely unknown entity, which is supposed to exist somehow and somewhere in a space unknown to us as well, and what is more, upon closer examination, possesses all the properties of absolute, metaphysical being. Ultimately, it is not clear why one does not simply attribute those properties that absolute space itself, rather than to the

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[11] For the temporal part see below section 3.4.

Alpha Body: “Here [. . .] we have moved in a circle; thought by its inner necessity has led us back to that very starting-point at which the first doubt and suspicion arose regarding the formulation of mechanical principles” (Cassirer 1910, 240; tr. 181).

### *3.2 Mach and the Sky of the Fixed Stars*

It was Ernst Mach (1872) the first to react to Neumann’s proposal. Mach agreed with Neumann’s assessment of the problem. According to Mach, however, that we do not need to postulate the existence of an unobservable body. Sensible experience itself unambiguously imposes it on us. Unbeknownst of Berkeley, Mach suggests that, when we observe a body moving of inertial motion we find that it moves linearly and uniformly with respect to the fixed stars. This is an empirically observable fact—it is the only thing we truly know from experience about the behavior of moving bodies (Cassirer 1910, 230; tr. 173). According to Mach, pondering the behavior of moving bodies in the absence of fixed stars—such as considering their hypothetical annihilation at a certain moment—is entirely futile exercise. As Mach (1883, 216) famously claimed, the world is not given to us twice: once in reality and once in thought, once in which the earth rotate with respect to fixed stars and once in which the fixed stars rotate with respect to earth. Rather, we must accept it as it presents itself to us in experience, without speculating on how it might appear under different conditions that we logically contrive (Cassirer 1910, 230; tr. 174). One could, of course, ask what laws of motion would apply if the fixed stars did not exist, or if we were deprived of the ability to orient our observations by them. However, we lack any possibility of judgment in this case.

According to Cassirer, “[i]n this solution of the problem offered by Mach, the consequence of the empiristic view is drawn with great energy” (Cassirer 1910, 230; tr. 174). Every scientifically allowable judgment derives its meaning only by relating to a tangible, presently existing reality, that is revealed to us to us by sensation; it cannot go beyond sensation and consider purely possible, not yet realized scenarios in its reasoning (Cassirer 1910, 230; tr. 174). However, to Cassirer it appears obvious that “this inference, though unavoidable from the presupposition [Mach] assumed, contradicts the known fact of scientific procedure itself” (Cassirer 1910, 231; tr. 174). The fundamental laws of physics consistently refer to cases that have never been given in experience, nor can they ever be given in it. Indeed, as Cassirer had shown extensively in other sections of the book, physical theories describe the behavior of abstract systems; the “real object of perception is replaced by its ideal limit” (Cassirer 1910, 159–173; tr. 120–130) whose behavior depends only on few selected parameters. These ideal constructions defining what the phenomena *would have been* if no other parameters exerted an influence. Thus, by describing physical systems, physical theories always indirectly provide a counterfactual characterization of actual phenomena. After all, Cassirer continued, one needs, only to refer Mach’s emphasis on the importance of ‘thought experiments’ in the history of physics (Cassirer 1910, 233f.; tr. 176f.).

According to Cassirer, contrary to Mach’s claim, it is undoubted, that the logical content of the law of inertia would remain unchanged even if, in the course of experience, reasons were to realize the fixed stars are not perfectly at rest. The propositions of pure mechanics would lose none of their validity with this ‘discovery’. They would be fully retained in the new system of reference that we would then have to seek (Cassirer 1910; tr. 177). However, such a transfer to a different reference frame, even merely in thought, would be impossible if the law of inertia only reflected the relations of moving bodies relative to a particular empirical frame of reference: “If the truth of the law of inertia depended on the fixed stars as these definite physical individuals, then it would be logically unintelligible that we could ever think of dropping this connection and going over to another system of reference” (Cassirer 1910, 235; tr. 177). The principle of inertia would not be a universal law of nature, but a description of specific phenomena in the environment. The claim that a body moves on a straight line *relative to fixed star*, would be akin tho the observation that sound travels, in all directions, at a fixed speed, *relative to the underlying air*.

Indeed, Mach coherently tried to develop precisely this line of reasoning. He suggested that one should “regard the fixed stars not as an element, which enters into the conceptual formulation of the law of inertia, but must conceive them as one of the *causal factors* on which the law of inertia is dependent” (Cassirer 1910, 233f.; tr. 177). Inertial motion is now *determined* (*bestimmt*) by the fixed stars and it is not simply *referred* to them. For Newton the plane in which the Foucault pendulum swings remains fixed in absolute space, for Mach the fixed hold or guide the plane of the pendulum. However, this theory, according to Cassirer, is not only *physically* implausible, but *epistemologically* misleading (Cassirer 1910, 233f.; tr. 177f.). Indeed Mach does not account for the physical meaning of the principle of inertia as a “universal principle of the phenomena of motion in general” by in indicating a suitable reference frame. On the contrary, he deprived the principle of inertia of the very status of a universal principle (Cassirer 1910, 233; tr. 177) and transformed it into “an assertion concerning definite properties and ‘reactions’ of a given empirical system of objects” (Cassirer 1910, 234; tr. 177), the moving bodies and a specific individual thing the sky of the fixed stars.

### *3.3 Streintz’s Fundamental Bodies*

Ultimately, in Cassirer’s assessment, Neumann’s and Mach’s apparently opposite proposal seems to share the same shortcomings: the principle of inertia has meaning only with reference to an *individual existing body*, whether it is unobservable or observable. If that body did not exist that the very notion of inertial trajectory would have no meaning. However, Cassirer objected, in this case we would search for a different coordinate system to refer the principle of inertia, rather than abandoning the latter. Indeed, Neumann himself conceded that, in principle, any given Alpha Body can be substituted by another, provided that the new body maintains a state of uniform rectilinear motion relative to the first (Neumann 1870, 21). According to Cassirer, it was Neumann’s former student, the Austrian Physicists Heinrich Streintz (1883) who attempted to address this issue. Streintz substituted the single body respect to which the principle of inertia holds with a *family of bodies* that satisfy certain empirical criteria: to perform no rotational motion and to be subjected to no external force . Streintz designates a body thus characterized as a ‘Fundamental Body’ (*Fundamental-Körper*) (*FK*) and any coordinate system that is rigidly bound to a *FS* a ‘fundamental coordinate system’ (*FS*). All *FS*’s are physically indistinguishable from one another by Streintz’s criteria: “The principle of inertia, in particular, can now be expressed in the form, that every point left to itself moves in a straight line and with constant velocity with reference to this fundamental body” (Cassirer 1910, 236; tr. 178).

According to Cassirer, the limitations of Streintz’s proposal appear quite evident once one understands his philosophical agenda. Streintz’s intention was to show that the principle of inertia is nothing more than the result of induction from observations that happen to hold true for certain individual bodies with specific physical properties, and which we have then assumed to be probable for all bodies of the same kind (Cassirer 1910, 237; tr. **). However, in Cassirer's analysis, Streintz's approach rests "on a conversion of the real logical and historical[12] relation" (Cassirer 1910, 236f.; tr. 178). The *FK*'s and the *FS*'s attached to them can never be found as 'empirical facts' (*empirische Fakta*) if the significance of both had not been determined beforehand in an 'ideal construction' (*ideeller Konstruktion*): "The seemingly pure inductions, which Streintz makes the basis of his explanation, are already guided and dominated by the fundamental conceptions of analytic mechanics" (Cassirer 1910, 237; tr. 179) that he prentends to justify.

The absence of rotational motion and the independence from any external force constitute the empirical criteria by which we recognize whether a given body can be regarded as a *FK*. However, Streintz has chosen those criteria under the assumption that the principle of inertia holds: "The *property* [*Merkmal*], by which we establish whether an individual case can be subsumed under a definite law, is logically strictly separated from the *conditions*, on which the validity of the law itself rests. The idea of inertia did not arise from" (Cassirer 1910, 237; tr. 179). The principle of inertia did not originate from observations of specific *FK*'s, that are non-rotating and not-subject to the influence of external forces; rather, it is only because the we assume the validity of the principle of inertia that we *search for* bodies of this kind and attribute to them the role of *FK*'s: "Thus the attempt of Streintz, in so far as it is meant to be a true founding of mechanics, involves a circle; for in the experiments and empirical propositions, which form the basis of it, there is already a tacit recognition of the principles which are to be deduced" (Cassirer 1910, 238; tr. 179).

### *3.4 Lange and the Notion of Inertial Frame*

For Cassirer, the early phase of the debate certainly had the merit of shifting the focus from the problem of absolute space to that of the 'reference frame'. However, in Cassirer's view, these early attempts to 'materialize the reference frame' were ultimately based on an epistemological fallacy: "It is not the *existence*" of a certain material frame (the Alpha Body, the fixed stars, the *FK*'s, etc.), "but the *assumption* of this existence, on which the validity of our mechanical concepts depends" (Cassirer 1910, 240; tr. 181). Whether or not this is the case does not add or subtract anything from the abstract formulation of the principle of inertia. In Cassirer's historical account, it was Ludwig Lange (1885, 1886b) who brought this point to light by attempting to address the problem of the reference frame without 'existential hypotheses' (*Existenzhypothese*) (Lange 1886b, 275).

As is well-known, Lange calls a system in which the principle of inertia holds an 'inertial system' (*Inertialsystem*): a reference system in which at least three points chosen *by convention* project non-collinearly from a point and move in a straight line. In this way, as already suggested by Neumann (1870, 16f.), we can also obtain an inertial measure for time, an 'inertial time scale' (*Inertialzeitskala*): the inertial unit of time is the time in which a free mass point that we choose *by convention* travels a certain distance in the inertial reference system. If the principle of inertia has to hold, then it must always be possible to construct a system in which arbitrarily many ($n > 3$) free particles move uniformly (Lange 1886b, 273f.). In this way, Lange stripped the

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[12] Cassirer often refers to Gallieo's expression *mente concipio* which in formulating the law of inertia, to indicate that is assumption that could not be directly verified.

Newton-Euler absolute space and absolute time of their superfluous elements. Newton and Euler implicitly held that, while inertial frames are empirically indistinguishable, they are not theoretically equivalent. They move with various, although unknown, uniform velocities relative to 'absolute space'. However, this assumption is not necessary for the construction of Newtonian mechanics.

In place of one absolute space and one absolute time, Lange showed that it is sufficient to introduce an equivalent class of possible inertial systems (moving at constant velocity relative to each other) with a global time scale. The problem of the *existence* of two metaphysical entities is transformed into the *role* that two conceptual constructions play within the entirety of classical mechanics:

> Thus there is no hypostatization of absolute space and absolute time into transcendent *things*; but at the same time both remain as pure *functions*, by means of which an exact *knowledge* of empirical reality is possible. The fixity, that we must ascribe to the original and unitary system of reference, is not a sensuous but a logical property; it means that we have established it as a *concept*, in order to regard it as identical and unchanging through all the transformations of calculation. [...] Only experience can ultimately decide whether this schema is applicable to the reality of physical thing and processes. Here also it is never possible to isolate the fundamental hypotheses and to point them out as valid individually in concrete perceptions; but we can always only justify them indirectly in the total system of connection that they effect among phenomena. We develop the determination of the 'inertial system' and the mathematical consequences connected with it purely in theory. In so far as any empirically given body seems to conform to these determinations, [...] we affirm that a material point left to itself must move uniformly in a straight line with reference to that body. (Cassirer 1910, 242; tr. 182)

Whether indeed such an inertial system exists cannot be decided *a priori*. However, that a system of bodies can serve as an inertial system (for example, the system of the fixed stars) does not signify "a *fact* that can be directly established by perception or measurement, but means that a *paradigm* is found here in the world of bodies for certain principles of pure mechanics, in which they can be, as it were, visibly demonstrated and represented" (Cassirer 1910, 242f.; tr. 183). For Cassirer, Lange's merit was to have clearly distinguished between the 'practical construction' of an *actual* reference system and the 'ideal construction' as the 'prototype' of all *possible* practical constructions (Lange 1902, 36).

Physical reference frames are for Lange only more or less successful concrete exhibitions of an ideal model, the 'inertial system', whose possibility is *required* by the principle of inertia. As a matter of fact, we know that "this requirement is never exactly fulfilled in experience, but always only with a certain approximation" (Cassirer 1910, 242; tr. 182). Just as there is no real straight line that fulfills all the properties of the pure geometric concept, there is no real body that corresponds in all respects to the mechanical definition of the inertial system. Therefore, the possibility always remains open to establish, "by the choice of a new point of reference, a closer and more exact agreement between the system of observations and the system of deductions" (Cassirer 1910, 242; tr. 183). Galileo initially took an Earth-fixed reference system as an inertial system in a first approximation. However, a solar-centered astronomical reference system proved superior because the fixed stars exhibit negligible motion relative to it. Yet, this system might ultimately prove inadequate and so on. Similar considerations apply to the measurement of time: "This relativity is indeed unavoidable; for it lies in the very concept of the object of experience. It is the expression of the necessary difference that remains between the exact conceptual laws we formulate and their empirical realization" (Cassirer 1910, 450; tr. 183).

Cassirer concedes that, when we base our statements about reality on free *ideal constructions* in this manner, it appears that a moment of 'arbitrariness' is allowed into our scientific consideration. In this sense, Lange characterizes "the concepts of the 'inertial system' and the 'inertial time-scale' [...] as mere *conventions*, which we introduce in order to survey the facts more easily, but which have no immediate objective correlate in empirical fact" (Cassirer 1910, 247; tr. 187). Lange's choice of the term 'convention'[13] is understandable as the recognition that scientific thought does not merely behave 'receptively', but displays a peculiar 'spontaneity' (Cassirer 1910, 248; tr. 187). Yet, Cassirer points out, "this self-activity [*Selbsttätigkeit*] is not unlimited and unrestrained" (Cassirer 1910, 248; tr. 187). Indeed, the choice of a convention considered in itself is indeed *contingent*; however, this contingency does mean arbitrariness. We prefer one definition over another, and not simply because of its 'convenience'; we favor one definition on account of its *necessity*, in achieving a certain 'goal': the notion of an inertial system is necessary for the very possibility of the principle of inertia, or better of the *leges motus* as a whole. The latter in turn are justified by the fact that serve as the basis for the formulation of a progressively more inclusive system of force laws. There is no external internal "criterion of 'objectivity'" beyond this process of convergence (Cassirer 1910, 248; tr. 187) .

With the transition from one absolute space to one class of equivalent inertial frames, Lange achieved a fundamental advancement over Euler. However, Cassirer suggests, that, by focusing on the problem of the 'reference frame', Lange unwittingly obscured Euler's fundamental point. The crucial issue is not the relation between Newton's laws of motion and the reference frame. Indeed, the main takeaway of Lange's analysis is that the validity of these laws *does not* depend on the choice of a particular inertial system. The essential concern is the relationship between Newton's laws of motion and the geometrical structure identifying inertial frames and trajectories that they presuppose.[14] The primary outcome of the nineteenth-century debate was to reveal that Newton's absolute space and time entail *more* structure than is strictly necessary for the possibility of Newtonian dynamics. However, a well-defined geometric framework is still needed for this purpose. In modern terms, the infinite collection of all inertial frames, along with their common time-scale, defines the three-plus-one chronogeometric structure that underlies classical mechanics. For Cassirer, the entire 'ontological' dispute between relativists and substantialists concerning the 'existence' of this structure misses the fundamental 'methodological' point. What is relevant is the role that such a structure plays within the system of Newtonian dynamics .

In Cassirer's interpretation, the choice of this particular structure cannot be determined by a purely logical argument, nor is it merely a generalization from experience. However, it is also not a matter of arbitrary convention, as Lange's terminology misleadingly suggests. The choice is constrained by the *function* that such a structure plays within the system of classical mechanics as a *whole*. In Cassirer's neo-Kantian language, this geometrical structure can be termed *a priori* because it plays the role of a necessary condition for the possibility of Newton's laws of motion, which, in turn, provides the necessary constraints for possible empirical force laws. In this sense, Cassirer is not afraid to suggest a comparison between emerging American pragmatism and critical philosophy (Cassirer 1910, 421–425; tr. 317–322). In both cases, "[t]he validity [*Geltung*] of a concept is determined by its performance [*Leistung*]" (Cassirer 1913, 37). However,

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[13] See DiSalle 1990.

[14] See, fn. 4.

contrary to pragmatism, critical philosophy provides a definite theoretical standard against which this performance can be measured: the ‘fact of science’ or, more precisely, the ‘fact of classical mechanics’.

## Conclusion: The Principle of Inertia and the Principle of Observability

After more than a decade as a *Privatdozent* in Berlin, in June 1919 Cassirer finally received the so-called *Ruf* at the recently established University of Hamburg. On November 6 1919, he held the inaugural lecture on the relationship between philosophy and exact sciences. Relying on material he had analyzed in his *Das Erkenntnisproblem*, Cassirer once again credited Euler for having reshaped these relationships in the midst of the Age of Reason. With respect to the question of the nature of space and time, the mathematical science of nature is the only authority we can rely upon (24–27). However, the trustworthiness of this authority was called into question a few days later. The London announcement of favorable results of Eddington’s eclipse observation on November 8 sanctioned the replacement of Newton’s theory of gravity by the theory of general relativity. The Archimedean point upon which critical philosophy relied, the ‘fact’ of Newtonian physics, no longer provides an absolutely firm foundation. Inevitably, Cassirer needed to take a stand. A few months after assuming his Hamburg professorship, Cassirer completed a book on relativity (Cassirer 1921) in the summer of 1920, and held a series of lectures on the topic in the following winter semester (Cassirer 1920–21; see also Cassirer 1920).

Cassirer continued to insist on the importance of philosophy maintaining a close relationship with the sciences. However, the ‘relativity revolution’ was a clear warning that philosophers should be careful to take science as a source of new questions rather than ready-made answers. In fact, “modern science gives us the exact opposite answer to the question of the true nature of space and time than mathematical physics gave us about 100 to 150 years ago!” (Cassirer 1920–21, 95). For Euler, space and time possess “absolutely certain physical reality” since they were necessary for formulating the laws of motion (95). However, at first sight, this answer appears to have become inadequate. Indeed, Einstein could now declare that, after general relativity, “space and time have been robbed of even the last vestige of physical objectivity [*Gegenständlichkeit*]” (95). This transformation was, of course, not accidental but occurred under the pressure of experience. However, to this pressure there must correspond a counter-pressure from the side of critical philosophy, until a new state of equilibrium is provisionally found: “Here we have again a dialectic within empirical science itself – which must become the awakener, the Paraclete of critical thinking” (96).

In the writings from the 1920s, relying on the same material he had addressed a decade earlier, Cassirer presents the history of spacetime theories as the interplay between the principle of inertia and the principle of observability (89–101). Leibniz embraced the principle of relativity motion as a consequence of the principle of observability. From a purely kinematic point of view, any arbitrary motion is empirically indistinguishable from stasis. It was Euler who showed that it is impossible to maintain the privileged status of uniform rectilinear motion required by the principle of inertia while maintaining a relativistic view of motion. However, when Euler concluded the existence of absolute space and time, he once again violated the principle of observability, introducing theoretical differences between ‘rectilinear uniform motion’ and stasis which have no empirical counterpart[15]

As we have seen, in Cassirer's reconstruction, a first balance between the two principles was reached only toward the end of the nineteenth century, when Lange introduced the notion of "'inertial system' that satisfied both requirements, those of the inertia principle and those of the '*principe de l'observabilité*' " (Cassirer 1921, 99; tr. 376). For Cassirer, special relativity—which he felt was premature to take into account in his 1910 book—fits quite well into this scheme. By abandoning the notion of absolute simultaneity, the theory extended the concept of 'inertial system' to account for the failure of 'ether-wind' experiments (Cassirer 1920–21, 101f.). Minkowski's (1909) four-dimensional formulation made fully perspicuous that special *relativity* actually makes a precise claim about the *absolute* geometrical structure of the 'world' (Cassirer 1921, 121; tr. 445f.).

Cassirer realized that general relativity represented, so to speak, a more significant *rupture épistémologique*. Due to the "*physical identity* of phenomena of inertia and weight" (Cassirer 1921, 66; tr. 401), it becomes impossible to empirically distinguish free-fall frames from inertial frames (Cassirer 1920–21, 103). By appealing to the principle of observability, Einstein elevated to the status of "a *principle*" the requirement that "for the physical description of the processes of nature, no particular reference body is to be privileged above any other" (Cassirer 1921, 44; tr. 383). Cassirer's formulation closely follows that of Einstein. However, precisely for this reason, it suffers from the same ambiguity. Indeed, as Hermann Weyl (1920) pointed out at around the same time, it is hard to see what a theory of motion satisfying this constraint could be like. If all coordinate systems are equally 'good', one could choose a coordinate system in which all time-like world-lines (four-dimensional trajectories of particles) become vertical straight lines, that is, they are all transformed at relative rest (Weyl 1924b). Thus, in such a theory, not only the absolute motion of a single body, but even the concept of relative motion of multiple bodies becomes meaningless (Weyl 1924a). If one takes the principle of relativity of arbitrary motion at face value, "no solution" to the problem of motion is possible (Weyl 1927, 74).

Like many of his contemporaries (see Reichenbach 1924), Cassirer did not initially fully grasp this point.[16] Only some decades later, he seems to have realized that also in the case of general relativity, one should "bring the law of inertia into agreement with the principle of observability" (Cassirer 1936, 156; tr. 126). Indeed, despite Einstein's philosophical inclinations, the final version of general relativity entails the conception of a privileged state of motion of bodies not subject to forces, just like any previous theory. As Cassirer seems to recognize, although rather in passing, Einstein's theory did not implement a 'generalized principle of relativity', but formulated a "'generalized law of inertia' which allows to combine in one expression the phenomena of inertia and gravitation. For every motion of a point under the influence of inertia and gravity the principle now holds that its world-line is always a 'geodesic' in the spacetime continuum" (Cassirer 1936, 63; tr. 50; translation modified). Einstein's theory did not achieve the relativity of arbitrary motion, but the relativity of the inertia/gravitation

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[15] It is interesting that Einstein criticized Cassirer's defense of Kant's doctrine of the ideality of space and time, on the basis of the fact that classical mechanics "demands an absolute (objective) space in order to assign real significance to acceleration, which Kant does not seem to have recognized" (Einstein to Cassirer, Jun. 5, 1920; CPAE, Vol. 10, Doc. 44). However, for Cassirer 'ideal' is not the opposite of 'objective'.

[16] Weyl (1924b) raises this objection against Schlick (1919).

split (Janssen 2014).

If Cassirer had expanded this point, he would come close to advocating a sort of neo-Kantian variant of 'inertial functionalism' (see Baker 2020).[17] Spacetime geometry must be evaluated because of its *function* within a coherent dynamic analysis of motion as a whole. Space-time must have *enough* structure to single out a preferred class of motions that define a standard of 'non-acceleration' (principle of inertia). However, this structure should not incorporate *redundant* features that lack empirical counterparts (principle of observability). As we now know, this formulation of the problem becomes fully clear only in four-dimensional formalism (Stein 1967). One might say that Leibniz's spacetime has *not enough* structure to define inertial motion, since it cannot distinguish straight and curved time-like worldlines; Newton's spacetime has *too much structure*, since it singles out absolute rest, a vertical straight worldline with respect to others (Ehlers 1973, 75). It was Lange who recognized that, in modern terms, what is needed is an 'affine' four-dimensional structure whose timelike straight world-lines represent free motions (75). In general relativity, the affine structure was not abolished, but, because of the empirical indistinguishability between free and free-falling motion, it became a dynamical field incorporating the effects of both gravitation and inertia on the motion of bodies (DiSalle 2006).

Cassirer conceded that, taken *logically*, the spacetime structure is not a discovery, but a choice. However, considered *functionally*, that is considering its 'role' within the rest of the theory's dynamics, the choice is *necessary.* It is required for the possibility of a feasible theory of motion, that is, a theory constrained by the principle of inertia on the one hand and the principle of observability on the other. In this sense, Cassirer could regard the spacetime structure as *a priori*, at least in the peculiar neo-Kantian sense of the expression. As his Yale student, Arthur Pap, pointed out, "Cassirer tends to assimilate Kant's doctrine of the *a priori* to the functional-pragmatic interpretation of the *a priori* " (Pap 1943, 455). Something is *a priori* because is hypothetically necessary, that is "functionally speaking", a necessary means for something else (449). In our case, a sufficiently rich spacetime structure is necessary for the possibility of a dynamical theory of motion, in which there is an objective difference between free motion and motion influenced by interactions. The history of spacetime theories from Leibniz to Einstein shows that fully relativistic theories of motion repeatedly failed. In Cassirer's language (Cassirer 1910, 355f.; tr. 269f.), one could then argue that the 'inertial structure' is a sort of 'invariant' in the historical succession of spacetime theories. As such, it can be provisionally considered a good candidate for the role of an *a priori* condition for the possibility of a theory of motion.[18] The necessity of an inertial structure is the true core of what Cassirer considered Euler's insight: "without the Newtonian concepts of an absolute space the law of inertia and, accordingly, the whole system of mechanics would become meaningless" (Cassirer 1943, 390)

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[17] See above, fn. 4.

[18] Should a Machian theory prove successful, Cassirer should concede that this provisional hypothesis is refuted.

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