Appearance and Reality: Einstein and the Early Debate on the Reality of Length Contraction

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In 1909, Ehrenfest published a note in the *Physikalische Zeitschrift* showing that a Born rigid cylinder could not be set into rotation without stresses, as elements of the circumference would be contracted but not the radius. Ignatowski and Varićak challenged Ehrenfest's result in the same journal, arguing that the stresses would emerge if length contraction were a real dynamical effect, as in Lorentz's theory. However, no stresses are expected to arise, according to Einstein's theory, where length contraction is only an apparent effect due to an arbitrary choice of clock synchronization. Ehrenfest and Einstein considered this line of reasoning dangerously misleading and took a public stance in the *Physikalische Zeitschrift*, countering that relativistic length contraction is both apparent and real. It is apparent since it disappears for the comoving observer, but it is also real since it can be experimentally verified. By drawing on his lesser-known private correspondence with Varićak, this paper shows how Einstein used the Ehrenfest paradox as a tool for an 'Einsteinian pedagogy.' Einstein's argumentative stance is contrasted with Bell's use of the Dewan-Beran thread-between-spaceships paradox to advocate for a 'Lorentzian pedagogy.' The paper concludes that the disagreement between the two ways of 'teaching special relativity' stems from divergent interpretations of philosophical categories such as 'reality' and 'appearance.'

Keywords: Albert Einstein • Paul Ehrenfest • length contraction • special relativity • Lorentzian pedagogy

Introduction

In the summer of 1909, in an attempt to construct a model of the 'electron,' Max Born (1909a, 1909b) put forward a relativistic definition of a rigid body for the case of linear acceleration. Born was confident that this result could easily be generalized (see Born to Ehrenfest, Mar. 17, 1909[1]). However, Paul Ehrenfest proved him wrong in the ensuing weeks. In a short note published in November in the *Physikalische Zeitschrift*, Ehrenfest (1909) argued that a Born rigid cylinder could not be given angular acceleration because, according to relativity theory, its circumference would contract but not the radius. At about the same time, Gustav Herglotz (1910) and Fritz Noether (1910) proved that a Born rigid body has only three degrees of freedom, instead of the six that a classical rigid body has. Einstein observed these developments with apparent detachment while being

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[1] Wheaton, 1977, 1, Sec. 9, Doc. 291.

absorbed in the project of a new theory of radiation (Einstein, 1909b, 1909a). However, despite attempts to solve the riddle (Born, 1910b, Planck, 1910), many believed that relativistic kinematics faced a crisis. The old concept of a rigid body had to be abandoned, and nothing suitable could be found to replace it (Abraham, 1910).

When the quarrel over the relativistic rigid body seemed to have reached a dead end, Vladimir Ignatowski (1910a) and Vladimir Varićak (1911c) entered the debate by challenging what became known as the 'Ehrenfest paradox' at its core. The entire controversy, they argued, was based on a conceptual misunderstanding. Ehrenfest's claim that it is impossible to set a rigid body into rotation is comprehensible from the point of view of Lorentz's theory, in which contraction is a *real* phenomenon. However, it is at odds with Einstein's theory, in which the contraction is only *apparent* and results from an arbitrary choice of clock synchronization. This argument was considered 'dangerously' misleading by both Ehrenfest (1910, 1911) and Einstein (1911d), who took a public stance and engaged in a somewhat heated debate in the pages of the *Physikalische Zeitschrift*.

Ehrenfest and Einstein were unlikely allies in the fight. Ehrenfest aimed to convince physicists to reject the new kinematics in the name of the rigid body; Einstein was ready to drop the idealization of the rigid body in the name of relativistic kinematics. However, both agreed on a central philosophical point: the dialectic between 'reality' and 'appearance' needs to be handled judiciously in a relativistic setting. Relativistic length contraction, they claimed, is a kinematic and perspectival effect and, in this sense, is only *apparent*. Nevertheless, length contraction is *real*, since it can in principle be ascertained empirically (Einstein to Lorentz, Jan. 23, 1915; CPAE, Vol. 8, Doc. 47). Ehrenfest's thought experiment illustrates this point. As the disk's angular velocity increases, so does the stress it experiences, eventually reaching a point where it should break (Einstein to Petzold, Aug. 19, 1919; CPAE, Vol. 8, Doc. 93). These effects would not manifest if the old kinematics held.

The early debate on the relativistic definition of a rigid body has been addressed in the historical literature, particularly by Arthur I. Miller (1981) in his classic monograph on special relativity (238–262). Moreover, Ehrenfest's rotating disk thought experiment has long been recognized by John Stachel (1989) as the 'missing link' in the chain of events culminating in general relativity. Giulio Maltese e Lucia Orlando (1995) suggested that Born's formalism for dealing with the relativistic definition of rigidity may have inspired Einstein's adoption of Riemannian geometry. However, since the appearance of these seminal works, additional documentary material has emerged that sheds new light on a previously neglected aspect of the early debate surrounding the Ehrenfest paradox.

The 10th volume of Einstein's *Collected Papers*, published in 2006, includes nine letters from Einstein to Varićak showing that, at that time, Einstein considered the issue of rigid rotation as the most interesting problem in the relativity debate (see Sauer, 2008). Moreover, Richard Staley (2008) has analyzed Ehrenfest's unpublished correspondence with Born and others, revealing that the latter's hostility towards the relativity principle persisted longer than previously believed. While the Ehrenfest paradox is known to have sparked a decades-long *technical* debate among physicists (see, *e.g.*, Rizzi and Ruggiero, 2004), these documents shed light on the fact that it also prompted the first *conceptual* debate on the reality of length contraction. The significance of this point becomes clearer when one recognizes the remarkable structural analogy between the dispute on the pages of the *Physikalische Zeitschrift* in the 1910s and a much more famous debate that erupted at the tables of the CERN canteen in the 1970s.

In a short, popular article, John Stewart Bell (1976) recounts raising a thought experiment to his colleagues that garnered much attention at the time (Dewan and Beran, 1959, Evett and Wangsness, 1960, Dewan, 1963, Evett, 1972). Two spaceships connected by a thread are moving with the same linear acceleration in a given inertial system. The distance between the corresponding points of the two spaceships remains constant. However, special relativity predicts that the thread will experience stresses that will eventually lead to its breakage. "Is it really so?", Bell asked his colleagues (Bell, 1976, 136). Based on Bell's version of events, "there emerged a clear consensus that the thread would *not* break!" (Bell, 1976, 136). According to Bell (1976), most physicists gave "this wrong answer" (136) because their training was based on Einstein's view that the length contraction is observer-dependent and is therefore only *apparent*. By contrast, those familiar with the Larmor-Lorentz-Poincaré approach "have stronger and sounder instincts" (Bell, 1976, 136), tending to conceive of the contraction as physically *real*, a modification of the molecular forces that keep the rod in equilibrium. From this perspective, it is more natural to expect the thread to break (Bell, 1976, 136).

While the debate in the *Physikalische Zeitschrift* has largely been forgotten, the discussion at the CERN canteen has often been a matter of contention (Brown and Pooley, 2001, see *e.g.*, 2006, Franklin, 2010, Fernflores, 2011), since the republication of Bell's (1976) short article in a best-selling collection of his philosophical papers (Bell, 1987, ch. 9). However, the two debates share a surprisingly similar structure. In both debates, the roles played by the Ehrenfest paradox and the Bell paradox are the same. The paradox arises in both scenarios due to the emergence of stresses in non–Born rigid motion. Despite their structural similarities, the two debates come to opposite conclusions. For Einstein, the emergence of relativistic stresses demonstrates that length contraction is a kinematic effect, *and nevertheless* it is real. By contrast, Bell saw the stresses as demonstrating that length contraction is a real effect, *and therefore* it requires a dynamic explanation (see also Bell and Weaire, 1992, 34).

The reason for the differences in the unfolding of the two debates can be traced to a fundamental disagreement about the underlying conceptual issue. Einstein and Ehrenfest challenged the implicit philosophical assumption on which Varićak's and Ignatowski's argument was based: the clear-cut opposition between 'kinematic = apparent' and 'dynamic = real.' According to Einstein, the contraction is 'apparent' because it disappears for the comoving observer, but it is also 'real' because it can be 'experimentally verified' (Einstein to Lorentz, Jan. 23, 1915; CPAE, Vol. 8, Doc. 47). By contrast, the physicists at CERN unknowingly took for granted Ignatowski and Varićak's assumption. The majority argued that stress effects would *not* emerge because relativistic length contraction is only 'apparent.' Bell, on the other hand, embraced the other prong of the argument and concluded that since relativity theory predicts that stresses occur, length contraction must be regarded as a physical, 'real' molecular contraction *à la* Lorentz.

Putting aside the contrast between the philosophy of physics and the history of physics, this is an example of philosophy's being at play *within* the history of physics. The divergent usage of established philosophical categories such as 'appearance' and 'reality' led to different implications regarding the proper way to 'teach special relativity.' Just as Bell famously used the thread-between-spaceships paradox to make a case for what he called 'Lorentzian pedagogy' (Bell, 1976, 77), Einstein used the rotating disk paradox as an instrument for what might be called an ‘Einsteinian pedagogy.’ The question of which pedagogical approach to prefer is still a subject of considerable debate. It remains an open question whether relativistic length contraction should be regarded as real or apparent, whether it should be presented as a dynamical or a kinematic effect (see, *e.g.*, Dieks, 1984, Brown, 2005, Janssen, 2009, D. J. Miller, 2010, Martinez, 2007, Redžić, 2008). This paper does not aim to settle the issue. However, it does hope to show that the reasoning that infers the dynamical nature of length contraction from the emergence of relativistic stresses (see, *e.g.*, Dieks, 1991, 254, 2004, 40) is not straightforward. Indeed, it is at least worth bearing in mind that Einstein bothered to take a stance against this argument repeatedly, both in private correspondence and in published writings.

1 Einstein and the Problem of Rigid Rotation in Relativity Theory

1.1 Ehrenfest and Born’s Relativistic Definition of a Rigid Body

On September 21, 1909, a young Einstein delivered his first major address at the scientific meeting of the *Gesellschaft Deutscher Naturforscher und Ärzte* in Salzburg, where he argued for a new theory of radiation.[2] The following day, Born (1909b) presented a simplified, geometrical version of his relativistic definition of rigidity[3] (see Born, 1909a). A bar in linear acceleration along the x-axis moves rigidly if its length r is constant as measured in its successive inertial rest frames.[4] In contrast to Newtonian physics, in relativity theory every part of the bar must undergo different rates of acceleration to remain rigid. Thus, rigidity is not a *property* of a rigid body but a *program* that involves applying forces to different parts of the object over time. However, Born’s definition captures the intuitive notion that a rigid body in motion remains free of strains because its changing acceleration maintains the body’s rest-frame dimensions.

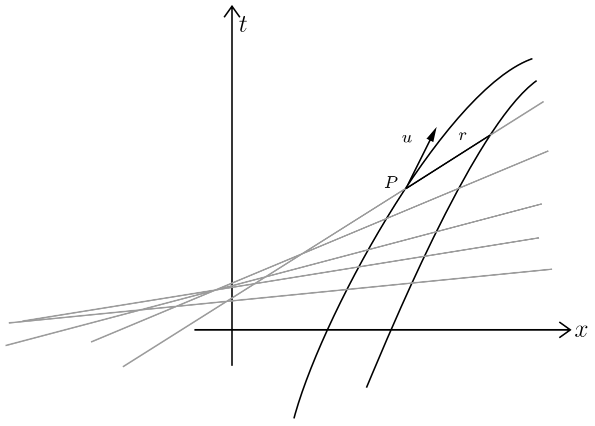

Figure 1: Adapted from Born, 1909b, fig. 2 and fig. 3

As Born later recounted (cf. Born, 1910a, 233fn. 2), Einstein immediately pointed out after the talk that a Born rigid body at rest could never be brought into uniform rotation. Indeed, just a few days after the Salzburg talk, Einstein made a similar remark in correspondence with Sommerfeld: “The treatment of the uniformly rotating rigid body seems to me of great importance because of an extension of the relativity principle to uniformly rotating systems according to analogous trains of thought as I did in the last § on the uni-

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[2] See McCormmach, 1970 for more detail.

[3] In pre-relativistic kinematics, the motion of a rigid bar in spacetime would appear like railway tracks of constant gauge and can be represented as being like railway tracks of constant gauge r. This definition is inconsistent with the theory of relativity, however, since it is based on an absolute notion of simultaneity. In relativity, a bar that appears rigid in one inertial frame may not appear rigid in another.

[4] See fig. 1. The unit four-vector tangent to the world-line of a moving point, *i.e.*, its four-velocity u, is used as the t'-axis of the comoving inertial frame. The x'-axis of this frame consists of points that are simultaneous with the worldpoint P that the four-velocity passes through. A body is considered rigid if it maintains the same length r in each of these instantaneous frames.

formly accelerated translation of my paper for the *Zeitschr[ift] f[ür] Radioaktivit*" (Einstein to Sommerfeld, Sep. 29, 1909; CPAE, Vol. 1, Doc. 179). As is well known, in his first review paper on relativity Einstein (1908) attempted to generalize the relativity principle to linear accelerating motion via the equivalence principle (§§17–19). Since a non-accelerated Cartesian coordinate system is nothing but a rigid body, Einstein wondered how such a reference body would behave when accelerating. He concluded that the acceleration-caused deformations could be neglected in the case of slow accelerations. Nonetheless, he may have realized that this expedient was not suitable for a rotating coordinate system, whose parts have unequal velocities.

Around the same time, Ehrenfest also came to a similar "non-Bornian" conclusion (Wheaton, 1977, m59, m60). On September 29, 1909, he sent a short note to the *Physikalische Zeitschrift*, in which he argued that Born's definition of a rigid body led to contradictions even for simple types of motion (Ehrenfest, 1909). Ehrenfest famously considered a Born rigid cylinder of radius R and height H rotating with angular velocity $\vec{\omega} < \frac{c}{R}$. Since radial line elements of the cylinder are perpendicular to the direction of motion, they would not be Lorentz contracted, and hence the radius R of the cylinder should remain unchanged. Therefore, the circumference U of the cylinder as calculated from the radius should be $U = 2\pi R$. However, since line elements along the rim are aligned with the direction of motion, they should be contracted by a factor of $\sqrt{1 - \frac{v^2}{c^2}}$, where $\vec{v_t} = R \times \vec{\omega}$ is the linear velocity of each element. As a consequence, the circumference of the cylinder, measured as the sum of the lengths of its various segments, turns out to be $U < 2\pi R$. The two results are incompatible, leading to an inconsistency in relativity theory.

Ehrenfest's (1909) note was published in November in the *Physikalische Zeitschrift*, alongside Born's (1909b) Salzburg paper. A few days later, Ehrenfest received a letter from his old friend Herglotz informing him that he had just finished a rather technical paper on the same topic, which was to be published in *Annalen der Physik* (Herglotz to Ehrenfest, Dec. 9, 1909; Wheaton, 1977, SC 5, Doc. 150). Herglotz's (1910) paper demonstrates that a Born rigid body has only three degrees of freedom, like a single particle, thereby confirming Ehrenfest's more qualitative argument (Herglotz, 1909). A student of Sommerfeld, Noether (1910) arrived at an equivalent result in the same weeks. Wilhelm Wien, who followed the debate closely as the editor of the *Annalen*, conceded to Sommerfeld that the relativistic rigid body had to be given up (Wien to Sommerfeld, Dec. 27, 1909).[5] However, Sommerfeld confessed that he and Noether had not found anything to replace it: "How a body really behaves (as a rigid body) and how an elastic body behaves, is still not clear to us" (Sommerfeld to Wien, Jan. 16, 1910).[6]

Despite the ongoing debate, Einstein informed Sommerfeld a few days later that he was not currently working on the "problem child [*Schmerzenskind*], the rigid body" (Einstein to Sommerfeld, Jan. 19, 1910; CPAE, Vol. 5, Doc. 197). Einstein believed that the available experimental data were insufficient to develop a theory of arbitrarily accelerated bodies since it was impossible to impart relativistic speeds to extended structures. One can only make assertions about systems undergoing infinitesimally slow linear acceleration. However, Einstein insisted again that it would have been desirable to "devise hypotheses about the

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[5] Archiv HS 1977-28/A,369, Deutsches Museum München.

[6] Sommerfeld, 2000–2004, Vol. 1, Doc. 164.

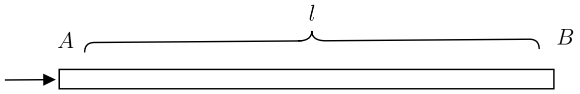

Figure 2: Adapted from CPAE, Vol. 5[10], Doc. 202b

behavior of rigid bodies that allow a uniform rotation" (Einstein to Sommerfeld, Jan. 19, 1910; CPAE, Vol. 5, Doc. 197). Unfortunately, his new position as Extraordinary Professor at the University of Zurich was taking up more of his time than he had anticipated. He admitted that his bad memory and lack of academic experience were to blame: "up to now I have only dealt with my subject as a dilettante" (Einstein to Sommerfeld, Jan. 19, 1910; CPAE, Vol. 5, Doc. 197).

1.2 Einstein, Varićak and the Rotating Disc

Einstein returned to the issue of rigid bodies about a month later, in correspondence with the Croatian mathematician Varićak (Einstein to Varićak, Feb. 15, 1910; CPAE, Vol. 5[10], Doc. 197a). Varićak was interested in the similarities between relativity theory and Lobachevski geometry (Varićak, 1911a, 1911b) and requested offprints of some of Einstein's papers. Einstein came to appreciate that "people with a great deal of mathematical education are now tackling the problem of relativity in order to shed light on its formal relationships" (Einstein to Varićak, Feb. 15, 1910; CPAE, Vol. 5[10], Doc. 197a). He admitted that Minkowski's work—which he had initially dismissed as unnecessary mathematical erudition[7]—had been of extraordinary value. However, at that time, he was not working on relativity but on a new theory of radiation (Einstein, 1909b) that he had just presented at the meeting of the *Gesellschaft Deutscher Naturforscher und Arzte* in Salzburg (Einstein, 1909a, see McCormmach, 1970).

Nonetheless, Einstein acknowledged that "[t]he most interesting problem that the theory of relativity currently offers seems to be that of the rotation of the rigid body (purely kinematical)" (Einstein to Varićak, Feb. 28, 1910; CPAE, Vol. 5[10], Doc. 197b). He pointed out that if one considers the Lorentz's contraction as the only contraction, "you run into contradictions, as Ehrenfest recently remarked in the *Physikalische Zeitschrift*" (Einstein to Varićak, Feb. 28, 1910; CPAE, Vol. 5[10], Doc. 197b). When writing to Varićak, Einstein had not yet received Herglotz's *Annalen* paper. However, he must have caught up with the literature soon after. A few weeks later, he informed his co-author Jakob Laub[8] that the "latest relativity-theoretical investigations by Born and von Herglotz" had shown that there is no such thing as "a 'rigid' body with 6 freedoms of motion in relativity" (Einstein to Laub, Mar. 16, 1910; CPAE, Vol. 5, Doc. 199). Consequently, Born's definition was considered unsatisfactory by most physicists in the field, and attempts were made to supply the missing degrees of freedom. However, it was deemed unlikely that the six degrees needed to produce the traditional rigid body kinematics in the low-speed limit could be recovered (Noether, 1910, 941f.).

When corresponding with Varićak again after his arrival in Prague at the beginning of April (Einstein to Varićak, Apr. 5, 1910; CPAE, Vol. 5[10], Doc. 202b), Einstein did not seem to be particularly concerned. It is probable that these results simply reinforced his belief that the idea of a rigid body was incompatible with relativity theory:

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[7] See Einstein and Laub, 1908b, 532.

[8] Einstein and Laub, 1908b, 1908a, 1909.

> First of all, it cannot be ruled out that the abstraction of the rigid body, which can be moved arbitrarily, does not fit into the theory of relativity at all. For example, consider the case of a rigid rod that is initially at rest and suspended freely in space, which suddenly receives an impulse at point A that lasts for an infinitely short time. As a result of this impulse, the end B can experience a change in position or gain speed only after the time $\frac{l}{c}$ has passed. Otherwise, 'superluminal signals' would exist, which leads to serious absurdities.[9] So, the rod is either deformed or only moves after a certain time due to the momentum. Both options are quite hazardous (the first one is also hazardous if examined closely). Therefore, it seems more sensible to do without the finitely extended rigid body altogether, especially if one only uses the infinitely small rigid body to define time and space. (Einstein to Varićak, Apr. 5, 1910; CPAE, Vol. 5[10], Doc. 202b)

In the early development of the new kinematics, a rigid body was used as the ideal reference frame. However, Einstein (1907) soon began to suspect that the notion of a rigid body was fraught with problems in relativity theory (see also Einstein to Sommerfeld, Jan. 14, 1908; CPAE, Vol. 5, Doc. 73). In particular, if a rigid body is pushed at one end, it cannot start moving at the other end immediately, since that would allow us to send a 'signal' at an infinite speed (Einstein, 1907, §3). In subsequent presentations of relativity, when introducing the notion of a *rigid* (*starr*) frame, Einstein (1908) remarked that it would have been more appropriate to speak of a *solid* (*fest*) frame that is not subjected to deforming forces (415; fn. 1). For the time being, Einstein seemed to be comfortable with the somewhat shaky notion of an infinitesimal measuring rod as a standard of length, that is, paradoxically, a rod whose length is negligible. The effects of acceleration on such rods could be ignored.[10]

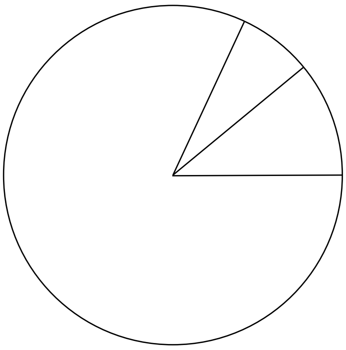
Figure 3: Adapted from CPAE, Vol. 5[10], Doc. 202b

In a letter that is no longer extant, Varićak must have attempted to persuade Einstein that the contradiction pointed out by Ehrenfest could be resolved by assuming that the radius R contracted by the same relativistic factor as the perimeter, thus preserving the ratio of 2π with the circumference.[11] Varićak proposed using light signals for the measurements, since they would move rectilinearly even in a rotating system (Einstein to Varićak, Apr. 11, 1910; CPAE, Vol. 5[10], Doc. 202b). The idea, we can presume,[12] was to send a light signal from the center of the disk towards L (as illustrated in fig. 4). When the disk is rotating with angular velocity $\vec{\omega}$ with respect to the non-accelerating system K, the signal will travel radially with a velocity component $\vec{v}_\perp = c$ until it reaches the point where the radius intersects the periphery. However, during this time the point will have moved, causing the observer at rest in K to see the signal travel along a curved path with a constant

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[9] In earlier work, Einstein (1907) showed that if superluminal signals were allowed, there would exist an inertial frame in which the arrival of the signal precedes its departure, *i.e.*, in which cause and effect are reversed (§3).

[10] See also fn. 22.

[11] A similar hypothesis has been explored by, *e.g.*, Grünbaum and Janis, 1977.

[12] Since Varićak's letters are no longer extant, the present reconstruction of Varićak's argument and Einstein's rebuttal is largely conjectural.

tangential velocity $\vec{\omega} \times R = \vec{v} = c$. According to Varićak's prediction, the light rays would intersect the circumference at an angle of $\vartheta \neq 0$ within L. Einstein informed Varićak that he had a similar idea, but instead of using light rays he considered the curvature of the radii of the solid material circle as seen from the rest system. However, he ultimately realized that this trick would not work since the radial and tangential lines would be orthogonal, as measured from an inertial frame momentarily moving with the same linear velocity as a point on the circumference (Einstein to Varićak, Apr. 5, 1910; CPAE, Vol. 5[10], Doc. 202a; see below).

It appears that Varićak was not convinced by the argument, and Einstein addressed the issue again in a subsequent letter just a few days later. First, he suggested that Varićak should have avoided the *dynamic* problem of the disk's behavior during the phase of angular acceleration. The latter involves even worse difficulties than the state of constant rotational velocity $\vec{\omega}$ (Einstein to Varićak, Apr. 11, 1910; CPAE, Vol. 5[10], Doc. 202b), since it depends on the disk's elastic properties. Unlike Ehrenfest, Einstein seemed to have been mainly interested in the *kinematic* problem of determining the shape of the disk rotating with constant angular velocity, as seen from the perspective of a non-rotating system (Einstein to Varićak, Apr. 11, 1910; CPAE, Vol. 5[10], Doc. 202b).[13] In relation to this latter issue, Einstein continued, in order to overcome Ehrenfest's objection, it was not enough to assume that the radii and the circumference are subjected to the Lorentz contraction. Rather, the contraction must apply to every material element of the rotating disk. However, as Einstein put it, "the fulfillment of this condition does not appear possible—this seems to have been proven by Herglotz in particular" (Einstein to Varićak, Apr. 11, 1910; CPAE, Vol. 5[10], Doc. 202b).

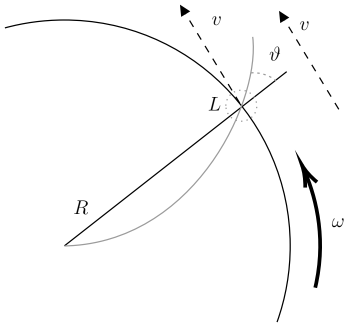

Figure 4: Adapted from CPAE, Vol. 5[10], Docs. 202a and b

To explain his objection to Varićak's proposal, Einstein returned once again to Varićak's thought experiment, delving into more detail. He asked Varićak to imagine radii engraved in a material circle before it is put into rotation (fig. 3).[14] The radial lines are everywhere orthogonal to the circles $R = \text{const}$, so that $\theta = 0$. Varićak had suggested that when the disk rotates with a constant angular velocity $\vec{\omega}$, the radius appears curved as judged from the coordinate system K at rest at the circle's center of rotation. Consequently, the radius and the circumference should intersect so that $\vartheta \neq 0$. To counter this conclusion, Einstein asked Varićak to consider a small spatial region L along the periphery of the disk. He then introduced an inertial frame K' that moves with a constant linear velocity $\vec{v} = \vec{\omega} \times R$ relative to K (Einstein to Varićak, Apr. 11, 1910; CPAE, Vol. 5[10], Doc. 202b).[15] At an

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[13] Relying on the equivalence principle, the latter could have been interpreted as a system at rest in a particular gravitational field.

[14] Einstein likely insisted on the materiality of the disk to counter Varićak's prejudice that optical lengths are less 'real' than material lengths; see below section 3.

[15] Einstein seems to use the approach that later has become standard of introducing a 'comoving inertial frame' K', in which each small element of the disc L is momentarily at relative rest (see, *e.g.*, Landau

instant in time, as judged from K', L has no translational velocity $\vec{v}$, but only a rotational velocity ω and acceleration $\omega^2 r$. Einstein doubted that these two effects combined would be enough to make $\vartheta' \neq 0$ as measured from K'. However, if $\vartheta' = 0$, then $\vartheta = 0$ as well: "Then Ehrenfest's point [*was Ehrenfest meint*] holds, which has been familiar to me for several years" (Einstein to Varićak, Apr. 23, 1910; CPAE, Vol. 5[10], Doc. 202b).

Regardless of what one thinks of Einstein's counterargument, the latter is revealing of Einstein's attitude towards the rotating disk thought experiment. Ehrenfest believed that his thought experiment should have convinced physicists to abandon the Lorentz-Einstein theory altogether, probably in favor of the ballistic theory of light developed by his friend Walther Ritz (1908). While Varićak attempted to come in support of relativity and circumvent Ehrenfest's objection, Einstein sided with Ehrenfest and embraced his result as yet another example of the unfeasibility of the notion of a rigid body in relativity theory. He conceded that Ehrenfest's reflections were probably incomplete, as Noether (1910) also pointed out. As Einstein concluded, however, "basically he's right" (Einstein to Varićak, Apr. 23, 1910; CPAE, Vol. 5[10], Doc. 202b). The absurdity of Ehrenfest's result shows that the disk cannot remain Born rigid if set into rotation but must experience deformations, depending on its elastic properties.

2 The Ehrenfest-Ignatowski Debate

In the meantime, attempts were made to address the problem of the missing degrees of freedom in relativistic rigid bodies. Born (1910a) acknowledged that a rigid body cannot rotate according to his first definition of rigidity, and he thus suggested an alternative definition of a rigid body with six degrees of freedom (Born, 1910b). However, Born had to concede that in a rotating rigid body so defined, the angular velocity measured by the system at rest decreases as the distance from the axis increases. Reacting to Born, Planck (1910) attempted to cut the Gordian Knot, arguing that in relativity theory only deformable bodies were allowed. The task of specifying the final state of a body set into rotation is a dynamical problem involving a yet-to-be developed relativistic theory of elastic media. For critics of relativity theory like Abraham (1910,), 531, however, the failure to recover a suitable concept of rigidity was a visible crack in the relativistic foundation, possibly a harbinger of impending collapse. Born's attempt at a relativistic definition of a rigid body must be viewed as a failure since it does not apply to rotational motion; Planck's appeal to relativistic elasticity theory was, for the time being, nothing more than wishful thinking: "Planck's remark cannot be seen as a weakening of Ehrenfest's objection" (Abraham, 1910, 531).

It is at this point that the Russian-born physicist Ignatowski, who was teaching at the Higher Technical School in Berlin at the time (Glick, 1987, 307), intervened in the discussion in an attempt to show that it was based on a misunderstanding. Around July 1910, Ignatowski likely sent Einstein the manuscript of a paper on relativistic rigid

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and Lifshitz, 1951, 43f. Møller, 1952, 223, Arzeliès, 1966, 204–206). The effects of angular velocity ω and acceleration $\omega^2 r$ are disregarded, and K' is treated as an inertial system moving with velocity $\vec{v} = \vec{\omega} \times R$ with respect to K. One can then apply the Lorentz transformations between K and K'. With respect to K, the radial element is not affected by the Lorentz contraction, resulting in $\theta = \theta' = 0$. On the contrary, the coordinate length of the tangential element is shorter. This difference gives rise to the conundrum pointed out by Ehrenfest. The sum of the lengths of its various segments appears $< 2\pi$ in K, but $= 2\pi$ in K'.

bodies to obtain his blessing (Ignatowski, 1910a). Einstein was critical, however: "With Ignatowski it is indeed as you suspected. He draws conclusions that contradict mine and then wants my approval" (Einstein to Hopf, Aug. 19, 1910; CPAE, Vol. 5, Doc. 221). He even planned to write a note on the topic for the *Annalen.* According to a later remark by Ehrenfest (1910), also Wien, the editor of the *Annalen*, warned Ignatowski against publication (413). Indeed, Ignatowski added an entire section (§6) to address Wien's concerns about his claim that superluminal velocities are allowed in special relativity.

Nevertheless, Ignatowski's (1910a) paper has the merit of presenting Herglotz's and Noether's technically demanding results in a simpler way (see Ishiwara, 1914, §15). Let us suppose there are two points on a body, P_1 and P_2, that are infinitesimally close to each other. dr^2 is the squared distance between them, measured synchronously. If the body is at rest, $dr = dr_0$, while if it is in motion, $dr = dr_1$, as measured at time $t_1 = \text{const.}$ in the comoving system. According to Ignatowski, the relativistic condition of rigidity can be expressed by the fact that for any two points on the body,

$$(dr_0)^2 = (dr_1')^2 \ , \tag{6}$$

that is, by the requirement that the shape of the body does not change in passing from rest to motion, and vice versa. If one requires that this is satisfied for all times t, one obtains the differential condition of rigidity:

$$\frac{d\,(dr_1')^2}{dt} = 0\,. \tag{7}$$

In eq. 13 of section §2 of his paper, Ignatowski reformulated the rigidity condition in the Lagrangian viewpoint used by Herglotz (1911), and in eq. 20 he presented it according to the Eulerian viewpoint used by Noether (1910). Ignatowski concluded that a body in rectilinear translation,[16] which is rigid according to eq. 20, can be brought to rest again (Ignatowski, 1910a, 620). By contrast, a uniformly rotating cylinder that satisfies eq. 20 cannot be brought to rest again while remaining rigid (Ignatowski, 1910a, 621). In particular, a rigid cylinder will "*apparently*" compress when set into rotation (Ignatowski, 1910a, 627; my emphasis). In the general case, bodies in relativity theory behave as if they were "a deformable medium, although only *apparently* deformable" (Ignatowski, 1910a, 626; my emphasis). Therefore, their behavior should be described by a relativistic theory of elasticity, as Planck (1910) had suggested (Ignatowski, 1910a, 627).

Ignatowski's insistence that the elastic behavior of bodies is only "apparent" (*scheinbar*) explains his attitude towards Ehrenfest's thought experiment. In a note added to the proof, Ignatowski wrote that he became aware of Ehrenfest's work by reading a paper by Stead and Donaldson (1910) in which the cylinder was replaced by a disk with an elastic membrane that can bend without resistance. If the disk is set into rotation, the membrane curves because of the Lorentz contraction and takes on a concave shape, assuming the form of a paraboloid of revolution. As Ignatowski commented:

> To my mind the whole thing seems to be a misunderstanding. Let's measure a line element along the circumference of the disk synchronously and sum over the circumference; we get a value smaller than $2\pi R$, where R means the radius of the disk. There is absolutely no contradiction in this, but everything is explained by the definition of a synchronous

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[16] The reference to 'rectilinear' motion turned out to be erroneous; see Ignatowski, 1911a.

> measurement. Mr. Ehrenfest's objection is nothing more than a confirmation that a uniform rotation satisfies condition eq. 20 § 2, and that accordingly a line element along the circumference of the cylinder measured synchronously appears shortened. In general, we can determine the true [*wahre*] form and dimension of a rigid body by measurement when and only when the body is at rest. Measurements on moving bodies yield only apparent [*scheinbar*] values. (Ignatowski, 1910a, 630)

According to Ignatowski, the difficulty denounced by Ehrenfest can be resolved without inconsistencies by using the relativistic definition of simultaneous measurement. In Ignatowski's view, measurements on a stationary disk reveal the true, or 'real,' geometric configuration of a body, while the apparent, or 'kinematic,' configuration of the body is only a result of Einstein's synchronization procedure. Changing the conventional definition of simultaneity does not alter the true form of the body in any way (see, below).

At the *Naturforschertersammlung* in Königsberg in September 1910, Ignatowski engaged in discussions with Sommerfeld and Born about the possibility of velocities greater than the velocity of light (Ignatowski, 1910b, see A. I. Miller, 1981, sec. 7.4.5). However, it was Ehrenfest (1910) who replied to Ignatowski's work on rigid bodies in "crushing detail" at the beginning of October (Klein, 1970, 153). Ehrenfest pointed out that Ignatowski's definition of rigidity as per eq. 6 was equivalent to Born's (1909a) first definition of rigidity, and eq. 7 was obtained by total differentiation. Eq. 13 and Eq. 20 were derived by mathematical manipulation, adding nothing to Herglotz's (1910) and Noether's (1910) results. In particular, Herglotz had already clarified that according to this definition of rigidity, a rigid disk could rotate with *constant* angular velocity. The main challenge lies exclusively in the question of the *transition* from rest to uniform rotation. If one accepts Born's definition of rigidity, as Ignatowski ultimately does, then the claim that no contradiction would emerge in the case of the rotating disk cannot be defended.

To clarify his point of view, Ehrenfest suggested the following thought experiment. A circular disk with equally spaced marks over its radius and circumference is provided. An observer B at rest relative to the disk records these marks of the stationary disk on a piece of tracing paper P. While the disk is rotating, at the moment his clock points to t, the stationary observer B holds a piece of tracing paper P_1 over it and traces all the marks on the rotating disk. Finally, the stationary observer B measures the mark distribution on the stationary tracing images Π and Π_1 on a piece of paper at rest (Ehrenfest, 1910, 1129).[17] Ehrenfest argued that the lengths of the periphery and radius of the disk represented in Π_1 are equivalent to the lengths that Ignatowsky referred to as being 'simultaneously measured' by the stationary observer at time t. He then posed the following two rhetorical questions to Ignatowsky:

> - Question I: Is the last assertion accurate? If not, then what distinguishes the result obtained by the observer at rest through 'synchronous measurement' of the rotating disk from the result obtained by measuring the stationary tracing image Π_1?
> - Question II: Assuming my assertion is valid, then the statements made by Mr. von Ignatowski regarding the 'synchronously measured' circumference and radius are not entirely consistent [*widerspruchslos*]. They correspond to the following statements regarding the tracing images: The tracing image Π_1 has the same radius as Π, but its

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[17] Ehrenfest's images on the tracing paper correspond to what John L. Synge (1956,), 120f. calls 'snapshots' and what Edward A. Milne (1935,), 107 calls 'world maps.' They correspond to hyperplane sections $t =$ const. of spacetime.

> circumference is shorter. How can we imagine tracing images with such properties without any contradiction [*widerspruchslos*]? (Ehrenfest, 1910, 1129)

The two images, Π and Π_1, show the same radius but different circumferences. This is unquestionably a contradiction, contrary to Ignatowski's claim. In a footnote, Ehrenfest pointed out that Ignatowski's reluctance to draw this conclusion was based on philosophical rather than technical reasons. Ignatowski believed that the synchronously measured distances in the rotating disk were merely *apparent* and that only measurements on the stationary disk were *real*. This terminology can be misleading, however: "It would be highly desirable, in case of further discussion, to avoid using the terms true and 'apparent' shape [*Gestalt*] of the rotating disk altogether, or, if this is not possible, to define the meaning of these terms with a simple and strict agreement" (Ehrenfest, 1910, 1129). It is quite likely that this final comment arose from correspondence with Einstein, who would make the same point a few months later.

In February, Ignatowski (1911d) put forth a relativistic theory of elasticity and calculated the change of the radius and the periphery of a disk with given elastic properties when set in rotational motion. Concluding the paper, he conceded Ehrenfest's point. Let the distance measured at rest between any two points on the disk be $dr = dr_0$, and let $dr = dr_1$ be the distance synchronously measured at time t_1, when the disk is in constant rotation. Then, $dr_1/dt = 0$, but $dr_0 \neq dr_1$. In other terms, the condition of Born rigidity can be fulfilled in the case of uniform rotation, but not in the case of the transition from rest to rotational motion. This remark, Ignatowski continued, "would have been sufficient to clarify the matter", without Ehrenfest's confrontational tone (Ignatowski, 1911d, 168). Ignatowski acknowledged that "the conclusion of §5 and the Note at the end of my paper has to be changed accordingly" (Ignatowski, 1911d, 168). Unlike Ehrenfest, however, Ignatowski did not conclude that relativity theory is inconsistent. Instead, he advocated dropping the concept of a relativistic rigid body: "We, therefore, agree with Mr. M. Planck that the deformation of a body is to be understood as an elastic problem" (Ignatowski, 1911a, 168f.). In fact, Ignatowski could already refer to a result published by Max Laue[18] (1911) in the February issue of the *Physikalische Zeitschrift*, which showed that in relativity theory, a body not only does not have six degrees of freedom but does not have any finite number of degrees of freedom as a direct consequence of c's being a limiting velocity.

However, Ehrenfest (1911) was not satisfied with Ignatowski's retraction and by March 1911 had drafted another "devastating attack" (Klein, 1970, 153). Understandably, Ignatowski refrained "from any further discussion because of the tone that Mr. P. Ehrenfest adopted towards me" (Ignatowski, 1911c, 607). Later in life, Ehrenfest regretted his dismissive attitude towards Ignatowski and even wished to exclude his two replies to Ignatowski from his collection of writings (Klein, 1970, 154). Nevertheless, Ehrenfest did point out the fundamental conceptual misunderstanding at the heart of Ignatowski's rather technical papers: the false opposition between 'real' and 'apparent.' Indeed, in a review paper on relativity published around that time, Ignatowski continued to consider the real length of an object, which is found by means of a synchronous measurement with the object at rest. If the object in question moves at a constant speed, "a synchronous measurement results in an *apparently* [*scheinbar*] shortened length of the object" (Ignatowski, 1911b, 5).

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[18] Max von Laue after 1913 when his his father was elevated to the nobility.

3 The Einstein-Varićak Controversy

A footnote in Ehrenfest's (1911) second response to Ignatowski (413; fn. 2) refers to a paper by Varićak (1911c). Ehrenfest planned to clarify Varićak's point in private correspondence. The paper, entitled "Zum Ehrenfestschen Paradoxon," was completed on February 5 and received by the *Physikalische Zeitschrift* on February 8, 1911. It is responsible for introducing the expression 'Ehrenfest paradox,' which is now commonly used in the literature. Varićak (1911c) responded to Ehrenfest's initial reply to Ignatowski by more explicitly raising questions about the nature of the contraction of moving bodies and its status as a real phenomenon. According to Varićak, the impossibility of setting a Born rigid body into rotation is understandable if one adheres to the Lorentz contraction hypothesis and views the contraction of moving rigid bodies in the direction of motion "as an objective change" (Varićak, 1911c, 169). In this view, every element of the periphery changes independently of the observer, while the elements of the radius remain non-contracted. However, the paradox dissolves if one adopts the "Einsteinian standpoint" (Varićak, 1911c, 169).

In this context, the contraction is only "an apparent, subjective phenomenon" fostered by the way we regulate clocks and measure lengths (Varićak, 1911c, 169). According to Varićak, Ehrenfest took the Lorentzian standpoint. For this reason, he concluded that the tracing images Π and Π_1 are different. If one adopts the relativistic standpoint, however, "those tracing images must be identical; they will have the same radius and the same periphery" (Varićak, 1911c, 169). If one performs measurement using light signals—by attaching mirrors at both ends of a rod and calculating the time light takes to go back and forth—it is clear why one would arrive at the idea of relativistic length contraction. However, one must always remain aware that from Einstein's perspective, the contraction "is only a psychological and not a physical fact, *i.e.*, the body has not really undergone any change" (Varićak, 1911c, 169). Indeed, the mechanical process of measuring by employing material measuring scales is different from using optical signals.[19]

To make his point, Varićak repeated Ehrenfest's thought experiment using rods moving uniformly with respect to one another. One should mark the rods on the tracing paper P, which is used to label the set of all events happening at the same time t for the rest system K, and then transfer these markings onto two pieces of tracing paper P' to create the two images Π and Π_1. These images represent an observer's instantaneous snapshot $t = t_0$ of the rod at the same instant in the stationary observer's inertial frame. One should then reproduce the two images Π and Π_1 on a non-transparent paper at rest: "I believe that the [stationary observer] will find the same distance both times, for in reality the rod has not become shorter" (Varićak, 1911c, 169). The reason for the difference is that the clocks at the points A and B of the moving rod, although they move concurrently, indicate different

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[19] Varićak (1911c,), 169 presented an additional argument to support the notion that relativistic contraction was not real, in contrast to the Lorentz-Fitzgerald contraction. The argument is quite revealing of his way of thinking. In 1902, Rayleigh (1902) raised the question of whether the Lorentz-Fitzgerald contraction would cause double refraction. According to Varićak (1911c,), 170, in a relativistic framework, this question would not even arise, as the contraction is merely apparent. On the contrary, Lord Rayleigh interpreted the contraction as a mechanical compression, similar to the compression experienced by a vertical piece of glass when a small amount of pressure is applied along its length. Such compression should result in the optical anisotropy of the glass, causing a ray of light incident upon it to split into two rays that follow different paths. The experiment conducted by Brace (1904) yielded negative results.

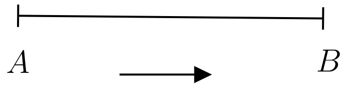

Figure 5: Adapted from CPAE, Vol. 5[10], Doc. 255a

times t' than the clocks of an observer at rest pointing to t. Thus, nothing happens to the rods themselves.

3.1 The Einstein-Varićak Correspondence

Varićak likely sent the draft of his paper to Ehrenfest, who showed interest in responding by the end of February (Varićak, 1911c). Einstein received the draft around the same time and reacted in a friendly way, but critically: "I do not agree with its content at all and I am quite sure that you are wrong. One must be very careful not to operate with the deceptive features 'real' and 'apparent' " (Einstein to Varićak, Feb. 24, 1911; CPAE, Vol. 5[10], Doc. 255a). Einstein explained his point by using a very simple argument. He considered a bar AB in parallel uniform translation along the abscissa of the rest system K (fig. 5). At a certain instant t, the end points A, B of the bar coincide with two points a, b of the abscissa of the rest system K. Varićak's "tracing paper experiment [*Pausversuch*]" (Einstein to Varićak, Feb. 24, 1911; CPAE, Vol. 5[10], Doc. 255a) captures a snapshot of these encounters.

Relativity theory predicts that the distance between a and b, as measured by a standard rod at rest in K, is shorter by a factor of $\sqrt{1-\frac{v^2}{c^2}}$ than the bar's length AB as measured by an identical rod placed at rest along the moving bar. Einstein emphasized that "[t]his follows necessarily from the transformation equations if one consistently interprets these times and coordinates physically. I do not understand how you arrive at the opposite view" (Einstein to Varićak, Feb. 24, 1911; CPAE, Vol. 5[10], Doc. 255a). To give a physical interpretation of the coordinate system, one must view the coordinate values as measurements obtained from stationary rods and clocks (cf. Einstein, 1910, 25f.; fn. 1). In the coming weeks, Einstein was set to relocate to Prague, where he had been appointed as the chair of theoretical physics at the German University. Given that it was relatively close to Zagreb, he was looking forward to meeting Varićak in person to discuss and clarify the matter at hand.

On March 1, 1911, Varićak's paper was published by the *Physikalische Zeitschrift*, and Einstein received a copy a few days later. He must have still been puzzled that a scholar of great mathematical sophistication such as Varićak could have found himself ensnared by such a fundamental conceptual misunderstanding. Thus, Einstein provided a more detailed explanation of his own point of view (Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a). His presentation of the issue is a compact version of similar presentations that can be found in Einstein's semi-popular writings of the time (Einstein, 1910, 1911a). He drew the diagram (reproduced in fig. 6) in which K is a non-accelerated system along whose x-axis a series of clocks $U_1, U_2, U_3, \ldots$ are placed and synchronized using light signals. A bar AB moves with uniform velocity v along the x-axis of K. By the length of AB we understand a number coordinated to it. In physics, there are two operations to assign that number:

1. a measuring unit rod is accelerated without changing its length until it attains the velocity v, i.e., until it is at relative rest with respect to the bar AB. The length

of AB is measured by successively applying the unit rod along the bar. Since the rod and the bar AB are at relative rest, the time needed to perform the operation is irrelevant. One lays the unit rod on AB so that one of its ends coincides with A and marks the bar at the position of the other end of the rod; the rod is then moved rigidly along the straight line prolongation so that its first end coincides with the mark of the second end. This process is repeated as often as necessary until the second end of the rod coincides with B. The number l of measuring rods that can be aligned in this way can be called the "'real' length" of AB,[20] or the "length measured from the bar itself" (Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a).

2. If procedure (1) were adopted to measure the length of the bar AB when the latter is moving with respect to K, the number obtained would not correspond to what we would naturally consider its 'length.' Indeed, if one places the first end of a unit rod at A at one time and then displaces it, the bar AB will have moved before we reach B at a different time. To avoid this, we need to ensure that we record the positions of A and B at the *same* time. To this end, a group of synchronized clocks $U_1, U_2, U_3, \dots$ are distributed along K. One marks two points a and b on K where we can find the two ends of the bar AB at the same instant t, as indicated by two clocks placed at a and b. One can then gently decelerate the unit rod from AB to K so that it remains identical to itself. The distance between a and b can be measured by successively applying the unit rod at rest along the line ab according to operation (1). The number l' of rods that fit between a and b is the length of AB as measured from the rest system K.

The results of both operations, (1) and (2), can equally be called the length of the bar AB. However, Einstein famously argued that there is no *a priori* reason for operations (1) and (2) to lead to the same *numerical value*. Classical kinematics assumed that $ab = AB$; that is, $l = l'$. Relativistic kinematics predicts that $ab < AB$; that is, $l' < l$ or $l' = l\sqrt{1 - \frac{v^2}{c^2}}$.[21] In principle, these predictions can be experimentally confirmed or disconfirmed by measuring the moving bar AB using a comoving unit rod, decelerating the latter while preserving its length[22] and subsequently measuring ab with the same unit

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[20] Of course, in general, the unit rod would not fit into AB an integral number of times; however, the measurement can be refined by using progressively smaller unit rods until one reaches the desired degree of approximation.

[21] See above fn. 23.

[22] The assumption that rods and clocks do not undergo any permanent changes if they are set into uniform motion and then brought back to rest is called the *boostability* of rods and clocks (Brown, 2005, 30). It is already implicit in Einstein's (1905) first relativity paper (897f.). However, Einstein made it explicit in his 1907/1908 review paper (Einstein, 1908, 410; fn. 1) and later in a semi-popular presentation of the theory (Einstein, 1910, 126; fn. 2), although only in a footnote. Einstein returned to this issue also during the discussion following his 1911 Zurich talk: "When the rod has completed its motion and returned, it has the same length. Similarly, the clock also has the same rate of ticking" (Einstein, 1911b, 44). To the best of my knowledge, it was Arnold Sommerfeld who first emphasized that special relativity presupposes a more general, and "unprovable" assumption (Blumenthal, 1913, 71): the rate of ideal clocks do not depend on their acceleration but solely on their instantaneous velocity (see Valente, 2013). In analogy with this 'clock hypothesis' a 'length' or 'rod hypothesis' can be formulated (see below fn. 35). Initially, Einstein seemed to have considered the independence of rods and clocks from their past

rod: “The contraction can be ascertained by measurement, *i.e.*, it is ‘real’” (Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a).

In relativity theory, the difference in judging the equality or non-equality of the lengths of the bar AB is a consequence of using different criteria to identify the points between which measurements should be performed. Due to the relativity of the “definition of simultaneity”, *identical* rods are used to measure the spatial distance between *different* pairs of points along the x-axis of different frames.[23] Like Ignatowski, Varićak seems to conflate relative and conventional. According to Einstein, the definition of simultaneity is indeed *relative*, that is, frame-dependent; contrary to Varićak’s claim, however, it is not of a “purely conventional nature” (Einstein, 1911a, 3–5), that is, it is not *arbitrary.*[24] As Einstein put it: “it is *impossible* to adjust the clocks in such a way that, even after the adjustment, the bar, if it has the speed $\pm v$ measured by the clocks, will always have the same length l' with respect to $[K]$” (Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a).

In Einstein’s view, Ehrenfest’s rotating disk gives a concrete representation of the fact that the contraction, in spite of being kinematic in nature, is physically real. The new kinematics does not allow for an increase in the angular velocity of the disk while keeping the rest length between neighboring points on the circumference constant.[25] As a consequence, tangential strains or deformations are induced in the disk purely from kinematic causes: “From this, one can conclude with Ehrenfest that a rotation *without elastic deformation* is impossible according to the theory of relativity if one accepts that a transverse shortening does not occur”[26] (Einstein to Varićak, Mar. 3, 1911; CPAE, Vol.

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non-inertial motions sufficiently obvious, as it was also implied in the old kinematics (Einstein to Besso, Oct. 31, 1916; CPAE, Vol. 8, Doc. 270). After Einstein’s (1918) debate with Hermann Weyl (1918a), the issue of the independence of prehistory of rods and clocks acquired a more prominent role in the former’s presentations of special relativity (Einstein, 1920b, [257]; fn. 19, 1922, 24, 39, 41; see also Einstein to Adler, Aug. 4, 1918; CPAE, Vol. 8, Doc. 1918). As Weyl (1918b,), 139 pointed out, this assumption needs a *dynamical* explanation. However, Weyl (1923,), 166 maintains that length contraction and time dilation are “velocity perspective”, that is, *kinematical*, effects. In my view, this was ultimately also Einstein’s stance (Einstein, 1922, 39). See Giovanelli, 2014 for more detail.

[23] In physics, points can be labeled with the aid of co-ordinate axes. The length of the moving bar AB, placed along the x-axis of the comoving system K', is the distance $l' = x'_b - x'_a$ (also called ‘proper length’). The length of the moving bar AB given in terms of the variables of the rest system K is the distance $l = x_b - x_a$ measured at the same instant t (the ‘coordinate length’). One can measure the value of l' with a rod at rest in K'; then, by setting $t_a = t_b$ and using the first Lorentz transformation, one obtains $x_b - x_a = \sqrt{1 - \frac{v^2}{c^2}}(x'_b - x'_a)$. In classical kinematics, one expected $x_b - x_a = x'_b - x'_a$. These predictions can be verified experimentally by measuring time and space coordinates with rods and clocks at rest in K.

[24] The confusion about Einstein’s argument may have stemmed from his claim that the synchronization of clocks *can* be achieved by establishing “*by definition*” that the one-way speed of light is equal to the two-way speed (Einstein, 1905, 894) However, this argument seems to pertain to the *pars destruens* of Einstein’s 1905 paper, where he aimed to expose the prejudice implicit in the old kinematics (se also fn. 33). Once the *pars construens* has been carried out—that is, once the new kinematics has been established—Einstein consistently maintained that relativistic kinematics is either true or false when interpreting coordinates as readings on rods and clocks. For example, see Einstein to Adler, Aug. 4, 1918; CPAE, Vol. 8, Doc. 1918 or Einstein, 1949, 57.

[25] The standard ‘kinematical’ explanation of this impossibility has been provided by Grøn (1975,), sec. IV. Due to the relativity of simultaneity, in a Born rigid increase of the angular velocity of the disk, a given point would be accelerated at the same time before and after itself.

[26] See above sec. 1.2.

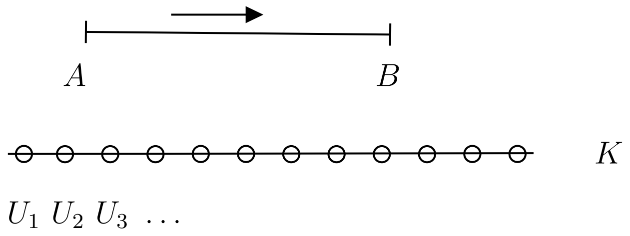

Figure 6: Adapted from CPAE, Vol. 5[10], Doc. 257a

5[10], Doc. 257a). Since the material of the disk resists deformation, the variation of angular acceleration must induce stresses within the disk's material. Indeed, at about the same time Herglotz (1911) developed a relativistic theory of elasticity based on the assumption that stresses arise when the condition of Born rigidity is violated. This is a result that is in principle empirically testable. If the old kinematics were applicable, an equivalent acceleration program would not give rise to any tangential stresses within the disk.

In Einstein's theory, length contraction is a *kinematic* effect that depends on the definition of simultaneity; however, it is just as real as length contraction in Lorentz's theory, where it is conceived as a *dynamic* effect due to the motion of a rod through the ether. The two theories derive the same quantitative measure for the contraction through different routes. To explain this point, Einstein resorts to his beloved comparison between relativity theory and thermodynamics:

> One cannot ask whether the contraction should be understood as a consequence of the modification of molecular forces caused by motion or as a kinematic consequence arising from the foundations of the theory of relativity. Both points of view are justified. The latter corresponds roughly to Boltzmann's treatment of the dissociation of gases in terms of molecular theory, which is perfectly justified, although the dissociation laws can be derived from the second law without kinetics. There is no (principal) difference with regard to the result, but only with regard to the foundations on which the investigation is based. (Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a)

In thermodynamics, different *methods* can be used to achieve the same *result*, such as the same law of dissociation of gases. As usual, he found the analogy with thermodynamics to be didactically useful for contrasting relativity theory with Lorentz's theory.[27] Lorentz kept the old kinematics in place and explained the negative result of the Michelson-Morley experiment by assuming that the arm of the interferometer is contracted in the direction of motion, due to the modification of molecular forces caused by the motion through the ether. By contrast, the theory of relativity shows that the same contraction of moving bodies follows from the new kinematics, without the need to introduce specific hypotheses about the structure of matter. It is not the motion of the interferometer through the ether that determines the contraction, but rather the motion of the earth with respect to the reference frame of the sun.

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[27] Einstein used the same comparison during the discussion following a lecture in Zurich given in January 1911 and published in April (Einstein, 1911b, III); see (Giovanelli, 2020). Notice that Bell and Weaire (1992) also employs the same analogy, likely independently.

Einstein was concerned that the "note could cause confusion" and warned Varićak that he had to take a public stance since his silence "could be taken as consent" (Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a). In April, Einstein wrote to Ehrenfest, inviting him to comment on Varićak's paper: "A completely erroneous note by Varićak that concerns us both recently appeared in the *Physikalische Zeitschrift.* Would you like to write the answer? A brief reply is necessary to avoid confusion" (Einstein to Ehrenfest, Apr. 12, 1911; CPAE, Vol. 5, Doc. 264). Ehrenfest had already prepared a response to Varićak's paper at the beginning of April.[28] However, perhaps to avoid another unpleasant controversy, he declined to answer publicly. It was ultimately Einstein (1911d) who, in May 1911, wrote a rejoinder for the *Physikalische Zeitschrift*, also entitled "Zum Ehrenfestschen Paradoxon."

Ehrenfest and Einstein formed an unexpected alliance in their dispute with Varićak. Ehrenfest was ready to reject relativistic kinematics for the sake of the rigid body; Einstein was ready to give up the notion of the rigid body for the sake of the new kinematics. However, they both agreed that Varićak's objection against Ehrenfest's paper was based on a fundamental *conceptual* misunderstanding, on the misuse of the opposition between the notions of 'real' and 'apparent.' Einstein pointed this out in his short reply to Varićak:

> Recently V[ladimir] Varićak published in this journal some comments that should not go unanswered because they may cause confusion. The author unjustifiably perceived a difference between Lorentz's conception and mine *with regard to the physical facts.* The question whether the Lorentz contraction is real is misleading. It is not 'real' inasmuch as it does not exist for a moving observer; but it is 'real' i.e., in such a way that, in principle, it could be detected by physical means, for a non-comoving observer. This is just what Ehrenfest made clear in such an elegant way. (Einstein, 1911d, 509)

The rest of the paper does not refer to the Ehrenfest paradox. Based on Einstein's correspondence with Varićak, it appears that he viewed the Ehrenfest paradox as an 'elegant way' to demonstrate that a purely kinematic effect can be considered 'real' since it produces measurable stresses. However, since Varićak's paper discussed the reality of length contraction in the context of linear relative motion, Einstein only addressed Varićak's argument.

In the writings of those years, Einstein appears to have still been reluctant to embrace Minkowski's (1909) reduction of kinematics to geometry. Indeed, he presented the key result of relativity as the distinction between the geometric and the kinematic configuration of a body (Einstein, 1908, 1910, 1911a).[29] The *geometric configuration* of a body at rest in the moving system K' is the set of points of the comoving system K' with which the body coincides. The *kinematic configuration* of the same body moving relative to the rest system K is the set of points of K with which the figure coincides at moment t. Classical kinematics took for granted that the two configurations are *identical.* Relativistic kinematics predicts that this is not the case:[30]

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[28] See the entries of 2 and 3 April [15 and 16 April in the Gregorian calendar] 1911, 'Red Book,' NeLR, Ehrenfest Archive, Notebooks, ENB:4-06.

[29] In modern terms, the distinction between the proper and the coordinate shape of a body.

[30] The terms 'configuration' and 'shape' should ultimately be interpreted analytically. In his famous example, Einstein (1905,), 903 showed that the geometric configuration of a *rigid* sphere of radius R at

> We obtain the shape of a body moving relative to the system K by finding the points of K with which the material points of the moving body coincide at a specific time t of K. Since the concept of simultaneity with respect to K that is being used in this determination is completely defined, i.e., is defined in such a way that, on the basis of this definition, the simultaneity can, in principle, be established by experiment, the Lorentz contraction as well is observable in principle.
> Perhaps Mr. Varičak might admit—and thus in a way retract his assertion—that the Lorentz contraction is a 'subjective phenomenon.' But perhaps he might cling to the view that the Lorentz contraction has its roots solely in the arbitrary stipulations about the 'manner of our clock regulation and length measurement.' The following thought experiment shows the extent to which this view cannot be maintained. (Einstein, 1911d, 509)

Einstein reformulated the thought experiment that he had presented in his correspondence with Varićak by using two rods, AB and $A'B'$. In this way, Einstein devised an ingenious trick to define the simultaneity of spatially separated events in a given inertial frame without using synchronized clocks. Einstein's twin-rod thought experiment runs as follows.[31]

Let us consider two rods AB and $A'B'$ (fig. 7). When compared at rest in a non-accelerated coordinate system K, they have the same length $AB = A'B'$. The two rods are supposed to be capable of moving parallel to the x-axis of K, sliding alongside each other with constant velocities v and $-v_1$, which can be arbitrarily large.[32] AB moves in the positive direction of the x-axis, while $A'B'$ moves in the negative direction of the x-axis. Einstein allows AB and $A''B''$ to move past each other so that A coincides with A' and B coincides with B'. Since the length of a rod according to special relativity depends on the square of the velocity alone (that is, it does not depend on the direction), by symmetry AB and $A'B'$ must have the same length relative to K as well. Thus, the encounters of the two ends of AB with the two ends of $A'B'$ are established as simultaneous events relative to K, without using clocks.[33]

One can imagine a device that leaves a mark a on the x-axis of K where the left end-points A and A' meet, and a mark b where B and B' meet, without the need to keep track of time. The theory of relativity predicts that the distance between the two marks, a and b, will be shorter than the length $AB = A'B' > ab$, whereas the old theory predicts

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rest in the moving system K' is the set of points whose coordinates satisfy the equation $x^2 + y^2 + z^2 = R^2$. The values of these coordinates are measured by unit rigid rods at rest in K'. Relativity theory predicts that the kinematic shape of the same figure with respect to K at time t is the locus of all points that satisfy the equations $x^2/\sqrt{1 - v^2/c^2} + y^2 + z^2 = R^2$. Thus, the kinematic shape of the same *rigid* body corresponds to an ellipsoid of revolution. This prediction can be tested by measuring the values of x', y', z' with identical unit rods at rest in K'. It is worth noticing that in the 1905 paper, the body is considered 'rigid.' By contrast, in Lorentz's theory it is deemed to be 'deformable.' Relativity theory provides a new kinematics of the rigid body's parallel translation.

[31] The thought experiment is reported by Pauli, 1921, 557, Møller, 1952, 46, Bridgman, 1962a, 93, Arzeliès, 1966, 112, Rindler, 1982, 29. Further discussions can be found in Sears, 1969, Schwartz, 1971, Winnie, 1972.

[32] Einstein does not seem to specify that $|v| = |v_1|$. This requirement would presuppose the use of clocks that have already been synchronized, making his argument circular (see, *e.g.*, Bridgman, 1962b, 93). It seems plausible that Einstein was aware of this issue, since he explicitly writes to Varićak that "the speed $\pm v$ [is] measured by the clocks" (Einstein to Varićak, Mar. 3, 1911; CPAE, Vol. 5[10], Doc. 257a). See fn. 34.

[33] Using this procedure, clocks in K can be synchronized in principle. The twin-rod thought experiment suggests that Einstein did not consider the so-called 'Einstein synchronization' using light rays to be essential. See also Einstein, 1922, 19. However, cf. Ohanian, 2008, ch. 4.

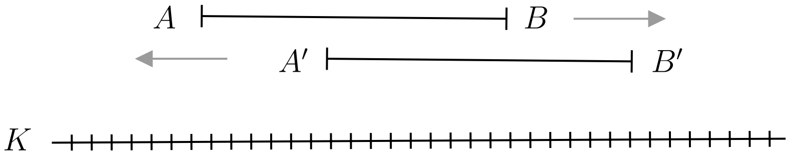

Figure 7

that $AB = A'B' = ab$. This prediction can be tested empirically[34] by gently decelerating one of the rods—say, AB—so that it does not change its length,[35] and laying it parallel to the x-axis along the two marks a and b in K. Thus, the contraction is *apparent*: it is not a property of AB or $A'B'$ taken by themselves but of their reciprocal relations.[36] However, it is also *real* since the prediction $ab < AB$ is an experimentally verifiable effect that, *pace* Varićak, an arbitrary resynchronization of clocks cannot eliminate.

Conclusion

By the spring of 1911, Einstein realized that his attempt at a new theory of radiation had failed (Einstein to Besso, May 13, 1911; CPAE, Vol. 5, Doc. 267; my emphasis). In a letter to Laub in the summer of 1911, Einstein only mentions the "relativistic treatment of gravitation" (Einstein to Laub, Aug. 10, 11; CPAE, Vol. 5, Doc. 275) as his current scientific interest (Einstein, 1911c). Ehrenfest visited Einstein in Prague at the end of February 1912. It is quite possible that they discussed Ehrenfest's reservations about Einstein's theory and his enduring support for Ritz's theory (see Ehrenfest, 1912).[37] What is clear is that in the following months, Einstein made the first published reference to Ehrenfest's thought experiment in a paper on gravitation published in February, where he pointed out that the geometry of the rotating disk is non-Euclidean (Einstein, 1912a, 356). Since a rotating system is equivalent to a system at rest in a suitable gravitational field, Einstein (1912b,), 1064 soon began to realize that the traditional physical interpretation of coordinates as readings on rods and clocks could not be maintained in the presence of gravitation (see Stachel, 1989, for more detail).

After returning to Zurich, Einstein famously found a solution to the conundrum with the help of his friend Marcel Grossman. However, his struggles with the meaning of coordinates in physics continued during the Berlin period (Giovanelli, 2021). In August 1915, while corresponding with Einstein about this issue, Lorentz returned, somewhat in

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[34] It is worth noting that Einstein does not need to provide a quantitative prediction, i.e., to claim that $ab = \alpha AB$. Thus, he does not need to assume that the speeds $|v| = |v_1|$ (see fn. 32). He only needs to show that classical and relativistic kinematics yield conflicting predictions.

[35] As mentioned in fn. 22, the assumption of rest length-preserving accelerations from one inertial frame to another is a special case of the 'rod hypothesis,' analogous to the 'clock hypothesis': at each time t, the readings of the accelerated rods and clocks always agree with the readings of a momentarily co-moving rods and clocks in inertial motion. Nevertheless, it is worth noticing that the *length hypothesis* presents greater challenges compared to the clock hypothesis due to the complexities involved in dealing with the acceleration of spatially extended structures in special relativity (see, *e.g.*, Redžić, 2008). Therefore, the *rod hypothesis* is typically applied to the nonuniform longitudinal motion of *infinitesimal* rods (Rindler, 1969, 54), a concept that remains somewhat unclear (see, for example, Synge, 1956, 32).

[36] In K the coincidences (A, A') and (B, B') occur simultaneously. However, by symmetry, in the rest frame of AB the coincidence (A, A') occurs before (B, B'); in the rest frame of $A'B'$, the opposite occurs.

[37] See also Einstein to Ehrenfest, Apr. 25, 1912; CPAE, Vol. 5, Doc. 384, Einstein to Ehrenfest, May 2, 1912; CPAE, Vol. 5, Doc. 390, Ehrenfest to Einstein, May 16, 1912; CPAE, Vol. 5, Doc. 394.

passing, to the question of the reality of length contraction (Lorentz to Einstein, Jan. 1, 1915; CPAE, Vol. 8, Doc. 43). Lorentz complained that, in a popular article, Einstein had referred to the Lorentz-Fitzgerald contraction as a "hypotheses invented *ad hoc*" (Einstein, 1915, 707) to neutralize Michelson's result (Lorentz to Einstein, Jan. 1, 1915; CPAE, Vol. 8, Doc. 43). Lorentz argued that such an objection might have applied to his first formulation of the contraction hypothesis. At a later stage, however, reacting to Poincaré's criticism, Lorentz provided a coherent theory of matter from which length contraction can be derived as a consequence. Lorentz regretted not having stressed this more, as it would have left less of an impression of being an *ad hoc* hypothesis (Lorentz to Einstein, Jan. 1, 1915; CPAE, Vol. 8, Doc. 43).

Lorentz argued that Einstein's approach was somewhat misleading from a "didactical" point of view (Lorentz to Einstein, Jan. 1, 1915; CPAE, Vol. 8, Doc. 43). If the contraction is derived as a consequence of the new kinematics "and nothing more is added in commentary", it could give rise to the suspicion that "only 'apparent' [*scheinbare*] things were involved here and not a real [*wirkliche*] physical phenomenon" (Lorentz to Einstein, Jan. 1, 1915; CPAE, Vol. 8, Doc. 43). Relativists often cannot avoid such ambiguity. In Lorentz's original theory, however, there is no doubt that the contraction is a 'real change' according to "common usage" of the expression and thus represents "a physical phenomenon" (Lorentz to Einstein, Jan. 1, 1915; CPAE, Vol. 8, Doc. 43). Lorentz added that the contraction of a rod moving with reference to the rest frame K is just as real as expansion at raised temperatures. In molecular theory, the expansion of a rod as a consequence of heat is explained in a way that is completely similar to the shortening of the rod as caused by the motion through the ether. Ultimately, the issue is a question that should be left to the theory of knowledge (see Lorentz, 1914, 23). However, Lorentz did not hide his inclination toward the dynamical approach.

Once again, Einstein replied by alluding to a more subtle dialectic between the real and the apparent:

> I ask you please to make allowances for my statements contained in *Kultur der Gegenwart*. Although I had 3 years of time to compose it, I had completely forgotten and was reminded of my commitment by Warburg one week before the delivery deadline. In this time I hastily pieced together the two articles as best I could. So please: do not punctiliously weigh every word!.[38] Regarding the erroneous view that the Lorentz contraction was 'merely apparent,' [*scheinbar*] I am not free from guilt, without ever having myself lapsed into that error. It is real [*wirklich*], i.e., measurable with rods and clocks, and at the same time apparent [*scheinbar*] to the extent that it is not present for the co-moving observers.[39] (Einstein to Lorentz, Jan. 23, 1915; CPAE, Vol. 8, Doc. 47)

Although Einstein's remark was meant to be conciliatory, it is clear that he and Lorentz

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[38] In his more carefully crafted presentations, such as his 1920 Leiden talk, Einstein (1920a) did not complain about the *ad hoc* nature of the Lorentz contraction but criticized the 'conspiratorial' nature of Lorentz's theory. This theory postulates theoretical asymmetries that lack empirical counterparts (8f.). As Einstein once put it: "Should (God) nature really have put us into an ether storm and have arranged its laws so precisely that we would never notice the storm?" (Einstein, 1920b, [5]).

[39] The length contraction occurs because the coordinate length of a moving rod is shorter than the proper length of the same rod at rest (see fn. 23). The effect is 'apparent' as it disappears once the rods are compared side by side and found to be identical. The same can be said of time dilation. However, the fact that the latter effect is 'real' can be proven experimentally in a more straightforward way, for example by observing a purely transverse Doppler effect (Giuliani, 2013). According to classical kinematics, only a longitudinal Doppler effect is expected.

operated with different notions of what is 'real' and what is 'apparent.' According to Lorentz, the Michelson-Morley experiment proves that the arm of the interferometer moving longitudinally with respect to the ether frame K appears to be rigid, but in reality, is shortened because of its motion, just as a metal bar is shortened because of the cold (see Lorentz, 1916, 196)[40] . However, Einstein's contemporary didactic accounts of the Michelson-Morley experiment reveal that he defended the opposite view explicitly. According to Einstein, "for a coordinate system $[K']$ moving with the Earth," the arm of the interferometer moving parallel to the direction of motion "is *not* shortened, but it is shortened for a coordinate system $[K]$ which is at rest relative to the sun" (Einstein, 1917, 28; my emphasis). From the perspective of K, the arm of the interferometer appears to be contracted, although nothing happens to it. This perspectival effect necessarily follows from the new kinematics "without the introduction of particular hypotheses" about the structure of matter (Einstein, 1917, 28). The effect is nevertheless real because it is empirically testable.

Lorentz and Einstein's rapid exchange on their opposed 'pedagogies' is the final chapter of a debate that, as we have seen, was occasioned by the Ehrenfest paradox at the turn of 1910. As this paper has demonstrated, Ignatowski and Varićak argued that the paradox disappears once one realizes that the Einstein contraction is purely apparent, since it is only the result of our way to synchronize clocks. Only a 'real' Lorentz contraction as a dynamic effect would cause the emergence of stresses when trying to set the disk into rigid rotation. However, for Ehrenfest and Einstein this conclusion was based on a profound misunderstanding. They argued that the contraction is indeed a kinematic effect, and thus only 'apparent,' but it is nevertheless real precisely because it produces dynamical stresses in a Born rigid rotating disk. When Einstein (1916, 1917) returned to the rotating disk thought experiment, the polemic had long been forgotten. Einstein was not interested in the *dynamic* problem of setting a Born rigid disk into rotation but in the *kinematic* image of the rotating disk in the coordinate system as judged from the rest frame (see Grøn, 2004, 288f.). The latter, through the equivalence principle, can be used to predict the behavior of rods and clocks in a gravitational field. Thus, Einstein's interest in the rotating disk thought experiment has mainly been studied as a fundamental step towards overcoming the metrical meaning of coordinates (Stachel, 1989).

As the old saying goes, however, history tends to repeat itself. The controversy about the reality of the length contraction that was settled in the 1910s on the pages of the *Physikalische Zeitschrift* erupted again in the 1970s at the tables of the CERN canteen. The case of two spaceships connected by a thread replaced that of the rotating disk. In both instances, however, the emergence of relativistic stress effects due to the violation of Born rigidity was a matter of contention. Yet if we trust Bell's recollections, the argumentative strategy of the CERN physicists surprisingly resembled that of Ignatowski and Varićak rather than that of Ehrenfest and Einstein. Most physicists involved in the discussion believed that the thread linking the spaceships would remain stress-free due to

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[40] As Jon Dorling (1968) pointed out, in Lorentz theory, the arm of an identical interferometer at rest in the ether frame K appears also contracted (not expanded) as measured from Earth frame K'—provided that clocks at rest in K' are synchronized using Einstein's synchronization procedure. Therefore, in Lorentz theory, there is both a 'real' contraction due to motion through the ether and an 'apparent' or perspectival contraction due to clock synchronization. I would like to express my gratitude to one of the anonymous referees for bringing Dorling's paper to my attention. I surmise that, for Einstein, that dialectic real/apparent is symmetrical.

the apparent nature of length contraction. By contrast, Bell's accurate prediction that the thread would experience stresses led him to conclude that length contraction is a real and dynamic effect, that "can do physical damage" (Bell and Weaire, 1992, 34). As a result, Bell viewed the thread-between-spaceships thought experiment as a valuable tool for promoting a 'Lorentzian pedagogy.'

As this paper has attempted to show, this conclusion is compelling only if one accepts the philosophical intuitions that seemingly underpinned the CERN debate. Like Varićak and Ignatowski physicists at CERN seemed to take for granted the conceptual opposition between 'dynamical = real' and 'kinematical = apparent'.[41] As we have seen, according to Einstein and Ehrenfest, this alternative was ill posed. In particular, Einstein redefined more precisely the terms as 'experimentally testable = real' vs. 'frame-dependent = apparent.' From this point of view, length contraction can be both 'apparent,' that is a perspectival/kinematic effect, and 'real,' as it results in detectable stresses. No such stresses would emerge if the old kinematics were in place. This conceptual issue was of such importance to Einstein that he joined forces with Ehrenfest, who was a vocal critic of relativity at that time. In doing so, he transformed the paradox formulated by Ehrenfest to expose the inconsistency of relativity theory into an opportunity to promote an 'Einsteinian pedagogy.'

Abbreviations

CPAE	Einstein, Albert. 1987–. *The Collected Papers of Albert Einstein.* Edited by John Stachel et al. 15 vols. Princeton: Princeton University Press.

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[41] Bell is more careful in distinguishing between 'Lorentzian pedagogy' and 'Lorentzian philosophy' (Bell, 1976, 147), emphasizing that endorsing the former does not entail accepting the latter (see also Bell and Weaire, 1992, 35). This distinction appears somewhat muddled in philosophical literature, where Bell's approach serves as a paradigm for the dynamical interpretation of relativity (Brown, 2005). The claim that the Dewan-Beran-Bell paradox supports such an interpretation is disputed by Fernflores, 2011.

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