Cassirer and the Arithmetization of Physics

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The Form of Science: Philosophy of Science in neo-Kantianism – Vienna 17/12/2025

UNIVERSITÀ
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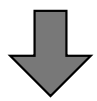
Introduction

- Klein (1895) argued that the nineteenth century witnessed a process of **arithmetization of mathematics**: mathematics sought to eliminate reliance on geometrical intuition and to ground itself in arithmetic constructions.
- Cassirer (1930s): the nineteenth and twentieth centuries witnessed a process of '**arithmetization of physics**': a shift from sensible qualities, through intuitive geometrical models, to abstract arithmetic formulas.



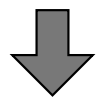
A through-line across the stages of Cassirer's philosophy of physics

Introduction

- **1910s:** energetics as the arithmetization of physics
- **early 1920s:** relativity as the arithmetization of geometry
- **mid 1920s–1930s:** arithmetization as a symbolic form
- **1930s–1940s:** arithmetization of mathematics and arithmetization of physics
- **1944:** arithmetization as a regulative ideal

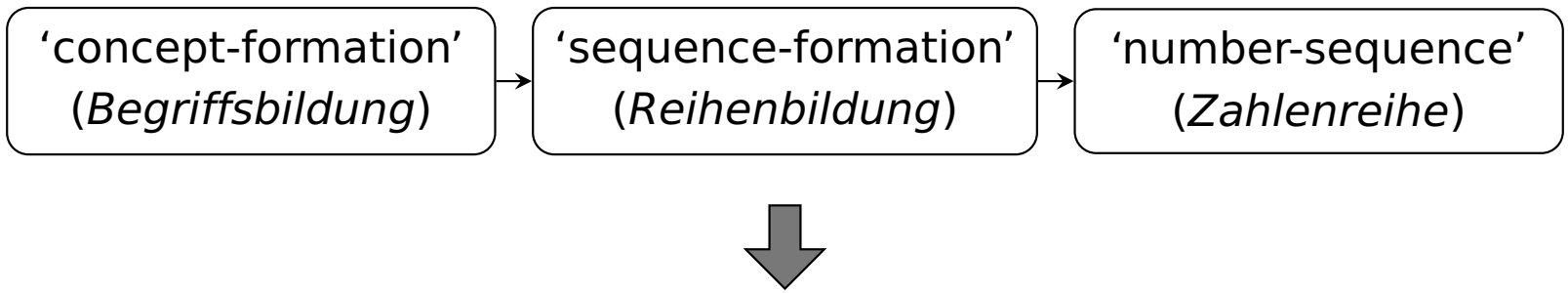
Introduction

- Cassirer's **historiography of physics**: arithmetization as a descriptive category
- Cassirer's **philosophy of physics**: arithmetization as a normative category



'concept-formation' (*Begriffsbildung*)

Introduction



- **structuralism**: each individual number is defined not by intrinsic content but solely by its relations within the numerical system;
- **constructivism**: these relations have the character of productive relations (*erzeugende Relationen*), exemplified by the successor function, which generates the sequence of all possible numbers.

Introduction

- **structural realism**: relations define relata (Klein's concept of group)[*]
- **structural constructivism**[†]: relations generate relata (Dedekind's concept of number)[‡]

natural science can grasp the real only by assigning to it a fixed place within the sequence of possible elements generated by a fixed principle

[*] Ihmig 1997; French 2014.

[†] Ferreirós 2023.

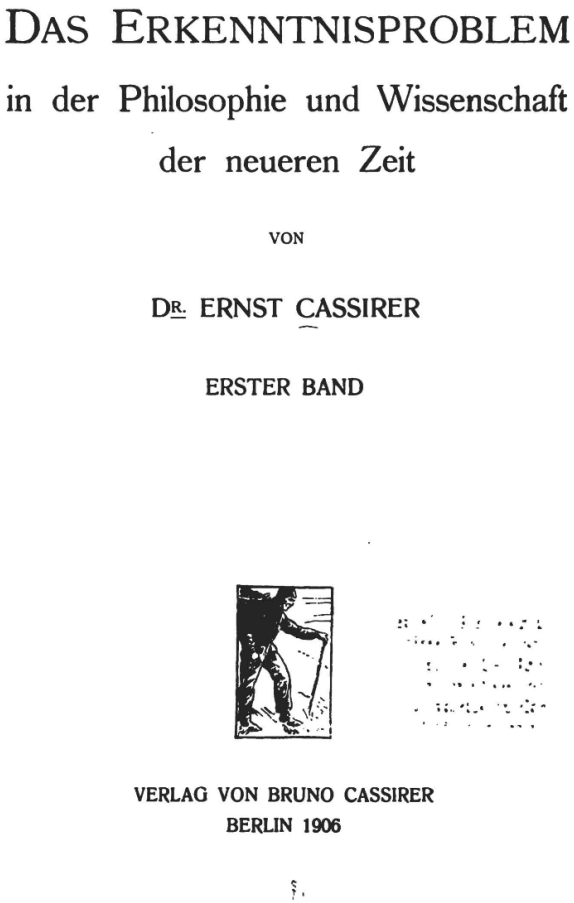
[‡] Reck 2020; Biagioli 2025; Koenig 2024.

Part II

The Energy Principle and Arithmetization of Physics

The Energy Principle and Arithmetization of Physics

Substance-Concepts vs. Function-Concepts. The Habilitation Lecture



DAS ERKENNTNISPROBLEM

in der Philosophie und Wissenschaft

der neueren Zeit

VON

DR. ERNST CASSIRER

ERSTER BAND

VERLAG VON BRUNO CASSIRER

BERLIN 1906

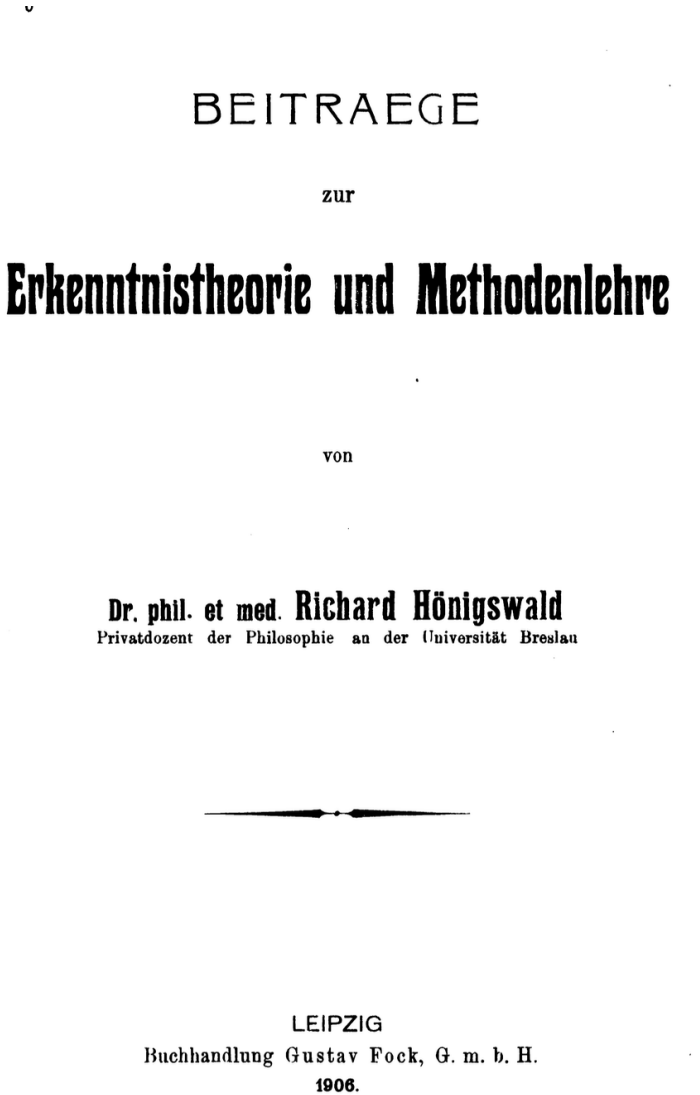
- **1906**: first volume *Das Erkenntnisproblem* as *Habilitationsschrift*
- **1906**: “Substanzbegriff und Funktionsbegriff” as *Habilitationsvortrag*

- ancient and medieval thought ⇒ **substance-concept** (*Substanzbegriff*)
- early modern mathematics (*Funktionsbegriff*) ⇒ **function-concept** (*Funktionsbegriff*)
- history of the mathematical science nature
  - the substance-concept ⇒ ‘concept of atom’ (*Atombegriff*) ⇒ mechanism
  - the function-concept ⇒ ‘concept of energy’ (*Energiebegriff*) ⇒ energetics

> A new and higher standpoint for assessing the general relation between the substance-concept and the function-concept is raised by modern energetics. [. . .] Robert Mayer already celebrates as the true logical advantage of the law of energy that it holds the mind directly to the observation of real physical processes and relieves it of all hypotheses concerning their inner 'grounds'. What is truly given to us—for example, in the transformation of heat into motion—are in fact only two qualitatively different processes, between which we nonetheless discover a constant quantitative relation of transition and thus a purely functional dependence. [. . .] The substantiality that it ascribes to force is therefore intended, by its very definition, to mean nothing other than a constancy of pure numerical relations [*Zahlenverhältnisse*]; a general quantitative lawfulness that spans all individual sequences of events and determines univocally [*eindeutig*] the transitions between them.
>
> *(Cassirer 1906b, 8 sq.)*

- **positivism**: energy not an independent substance, it signifies nothing but a functional relation among measurable quantities given in experience
- **neo-Kantianism**: positivism failed to recognize that such mathematical dependency is not derived from experience but rather constitutes a condition that makes experience possible

If no univocal quantitative coordination (*eindeutige Zuordnung*) between qualitatively different phenomena could be established, science would become *quiddam vagum et absonum* (Leibniz)



BEITRAEGE

zur

Erkenntnistheorie und Methodenlehre

von

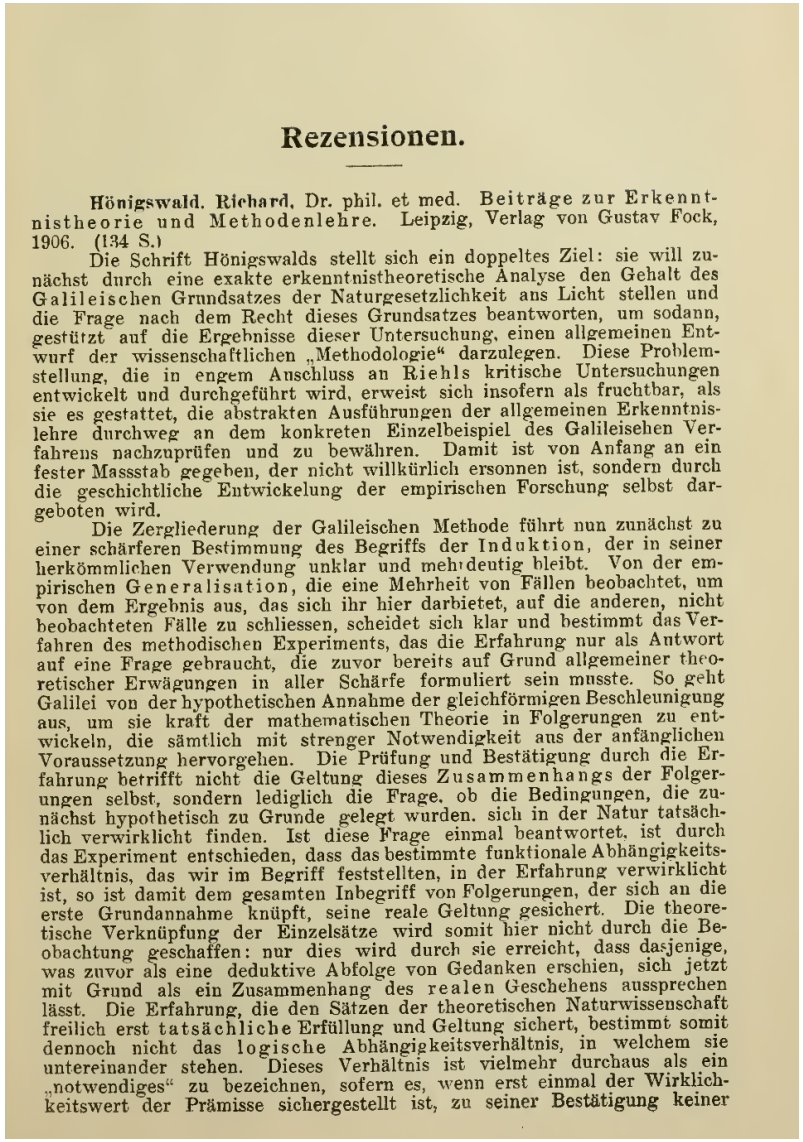
Dr. phil. et med. Richard Hönigswald
Privatdozent der Philosophie an der Universität Breslau

LEIPZIG
Buchhandlung Gustav Fock, G. m. b. H.
1906.

- from Galilei to Mayer $\Longrightarrow$ physics **describes** the relations among phenomena $\Rightarrow$ within experience
- from Descartes to Helmholtz $\Longrightarrow$ physics **explains** phenomena through the construction of pictures (*Bilder*) or models (*Modelle*) $\Rightarrow$ beyond experience

The Energy Principle and Arithmetization of Physics

Space-Concepts vs. Number-Concepts. The Hönigswald-Review



Rezensionen.

Hönigswald. Richard. Dr. phil. et med. Beiträge zur Erkenntnistheorie und Methodenlehre. Leipzig, Verlag von Gustav Fock, 1906. (134 S.)

Die Schrift Hönigswalds stellt sich ein doppeltes Ziel: sie will zunächst durch eine exakte erkenntnistheoretische Analyse den Gehalt des Galileischen Grundsatzes der Naturgesetzlichkeit ans Licht stellen und die Frage nach dem Recht dieses Grundsatzes beantworten, um sodann, gestützt auf die Ergebnisse dieser Untersuchung, einen allgemeinen Entwurf der wissenschaftlichen „Methodologie" darzulegen. Diese Problemstellung, die in engem Anschluss an Riehls kritische Untersuchungen entwickelt und durchgeführt wird, erweist sich insofern als fruchtbar, als sie es gestattet, die abstrakten Ausführungen der allgemeinen Erkenntnislehre durchweg an dem konkreten Einzelbeispiel des Galileisehen Verfahrens nachzuprüfen und zu bewähren. Damit ist von Anfang an ein fester Massstab gegeben, der nicht willkürlich ersonnen ist, sondern durch die geschichtliche Entwickelung der empirischen Forschung selbst dargeboten wird.

Die Zergliederung der Galileischen Methode führt nun zunächst zu einer schärferen Bestimmung des Begriffs der Induktion, der in seiner herkömmlichen Verwendung unklar und mehrdeutig bleibt. Von der empirischen Generalisation, die eine Mehrheit von Fällen beobachtet, um von dem Ergebnis aus, das sich ihr hier darbietet, auf die anderen, nicht beobachteten Fälle zu schliessen, scheidet sich klar und bestimmt das Verfahren des methodischen Experiments, das die Erfahrung nur als Antwort auf eine Frage gebraucht, die zuvor bereits auf Grund allgemeiner theoretischer Erwägungen in aller Schärfe formuliert sein musste. So geht Galilei von der hypothetischen Annahme der gleichförmigen Beschleunigung aus, um sie kraft der mathematischen Theorie in Folgerungen zu entwickeln, die sämtlich mit strenger Notwendigkeit aus der anfänglichen Voraussetzung hervorgehen. Die Prüfung und Bestätigung durch die Erfahrung betrifft nicht die Geltung dieses Zusammenhangs der Folgerungen selbst, sondern lediglich die Frage, ob die Bedingungen, die zunächst hypothetisch zu Grunde gelegt wurden, sich in der Natur tatsächlich verwirklicht finden. Ist diese Frage einmal beantwortet, ist durch das Experiment entschieden, dass das bestimmte funktionale Abhängigkeitsverhältnis, das wir im Begriff feststellten, in der Erfahrung verwirklicht ist, so ist damit dem gesamten Inbegriff von Folgerungen, der sich an die erste Grundannahme knüpft, seine reale Geltung gesichert. Die theoretische Verknüpfung der Einzelsätze wird somit hier nicht durch die Beobachtung geschaffen: nur dies wird durch sie erreicht, dass dasjenige, was zuvor als eine deduktive Abfolge von Gedanken erschien, sich jetzt mit Grund als ein Zusammenhang des realen Geschehens aussprechen lässt. Die Erfahrung, die den Sätzen der theoretischen Naturwissenschaft freilich erst tatsächliche Erfüllung und Geltung sichert, bestimmt somit dennoch nicht das logische Abhängigkeitsverhältnis, in welchem sie untereinander stehen. Dieses Verhältnis ist vielmehr durchaus als ein „notwendiges" zu bezeichnen, sofern es, wenn erst einmal der Wirklichkeitswert der Prämisse sichergestellt ist, zu seiner Bestätigung keiner

"differentiation of the methods of empirical knowledge itself" (Cassirer 1909b, 93)

- **arithmetic** method ⇒ number-concepts (*Zahlbegriff*)
- and the **geometric** method ⇒ space-concepts (*Raumbegriff*)

- **arithmetic approach** $\Longrightarrow$ ‘specification’
  - energetics: the ‘essence’ of the individual forms of energy is understood when the fixed *equivalence values* that determine the numerical relations between the different domains can be univocally (*eindeutig*) established (Cassirer 1909a, 93).
    - a quantity of heat corresponds to a definite quantity of mechanical motion
- **geometric approach** $\Longrightarrow$ ‘homogeneity’
  - mechanism attempts to explain all phenomena by providing a model: A it omits the nonessential features and focuses exclusively on the ‘spatial structure’.
    - heat is reducible to the microscopic motion of particles

> The problem of whether [. . .] we should content ourselves with relations or instead resort to intuitive models ultimately leads, [. . .] back to the simple opposition between arithmetic and geometric methodology. The determination of the 'laws' of appearances is carried out by establishing certain numerical relations [*Zahlenverhältnisse*]. [. . .] At this point [. . .] a new question indeed arises [. . .] namely, how to bring [the phenomena] together within a strictly unified and general plan. [. . .][T]is task, in order to be fulfilled, repeatedly calls forth the geometric 'model' and schema beyond the abstract functional relation. [. . .] Just as arithmetic represents the interest of 'specification,' so geometry here represents the interest of the 'homogeneity' of the explanatory grounds. [. . .] This connection is not guaranteed by number alone; rather, its establishment ultimately requires a reduction to space as the unified and homogeneous basic schema.

(Cassirer 1909a, 94)

The Energy Principle and Arithmetization of Physics

SUBSTANZBEGRIFF UND
FUNKTIONSBEGRIFF

Untersuchungen über die Grundfragen
der Erkenntniskritik

von

ERNST CASSIRER

VERLAG VON BRUNO CASSIRER
BERLIN 1910

Theory of science-formation in general

- **descriptive sciences** (botany, zoology): 'discursive concept-formation', based on the abstraction of identical characteristics that an actual manifold of different elements have in common ⇒ genus-concept (*Gattungsbegriff*)
- **mathematizing sciences**: 'constructive concept-formation', based on the production of a sequence (*Reihe*) of possibly different elements through the iterative application of an identical rule (Cassirer 1910, 196; tr. 1923, 147f.) ⇒ sequence-concept (*Reihenbegriffe*)

> "The requirement posed by this concept is fulfilled wherever the elements of a manifold are derived from specific sequence-principles [*Reihenprinzipien*] and are exhaustively characterized by them."
>
> *(Cassirer 1910, 131; tr. 1923, 99)*

Mathematical science formation

- **number-concept** (*Zahlbegriff*): the number-concept is paradigmatic: each individual number nothing defined than their positional value in a sequence and sequences are given by rules of generation
- **space-concept** (*Raumbegriff*): space-concept retains an irreducible intuitive–qualitative character that (points $\neq$ instants): a progressive "conversion of space-concepts into number-concepts" (Cassirer 1910, 93; tr. 1923, 71), that is, a transition from geometry to arithmetic[§].

[§] cf. Schiemer 2018.

scientific concept-formation

- **Mechanism**: “In order to advance from numbers to material physical existence, it is necessary to have the mediation of the space-concept [*Raumbegriff*]. Space itself is taken here, however, in a sense that virtually stamps it as the pure symbol of number” (Cassirer 1910, 206; tr. 1923, 156).
- “**Energetics** contains a motive from the beginning, which protects it more than any other physical view from the danger of an immediate hypostatization of abstract principles. Its fundamental thought, from an epistemological point of view, does not *go back primarily to the concept of space, but to that of number*” (Cassirer 1910, 251; tr. 1923, 189).

The Energy Principle and Arithmetization of Physics

- Physical concepts “appear to be so many means of grasping the ‘given’ in sequences, and of assigning it a fixed place within these **sequences**” (Cassirer 1910, 196; tr. 1923, 148).
- the embedding “of the sensuous manifold into sequences of purely mathematical structure remains inadequate as long as these sequences are **separated** from each other” (Cassirer 1910, 252; tr. 1923, 190).
- Scientific knowledge requires a principle that establishes a ‘**coordination**’ (*Zuordnung*) among the various numerical sequences: any change of state transformed into a standard change (the elevation of a given weight to a certain height) = ‘mechanical work’

The energy principle, the conservation of mechanical work, is the condition for the univocality of the coordination (*Eindeutigkeit der Zurodnung*) between different sequences of changes, which yield the same numerical value when expressed in work units.

The Energy Principle and Arithmetization of Physics

> The conceptual connection established by the equivalence values [*Äquivalenzwerte*] among the various sequences provides a logical bond no less firm than the reduction to a common mechanical model. The conceptual requirement of homogeneity is thus operative both in the energetic and in the mechanical interpretation of natural processes: the only difference is that, in the one case, its realization relies solely on the concept of number, whereas in the other it also demands the concept of space. The dispute between these two interpretations can ultimately be decided only by the history of physics itself; for only here can it become evident which mode of consideration is finally able to do fullest justice to the concrete tasks and problems. [. . .] A strictly 'hypothesis-free' account of natural processes cannot, of course, be achieved even along this path. For the translation into the language of abstract number-concepts [*Zahlbegriff*] entails no less than the translation into the language of space-concepts [*Raumbegriff*] a theoretical transformation of the empirical material of perception.

(Cassirer 1910, 206; tr. 1923, 156)

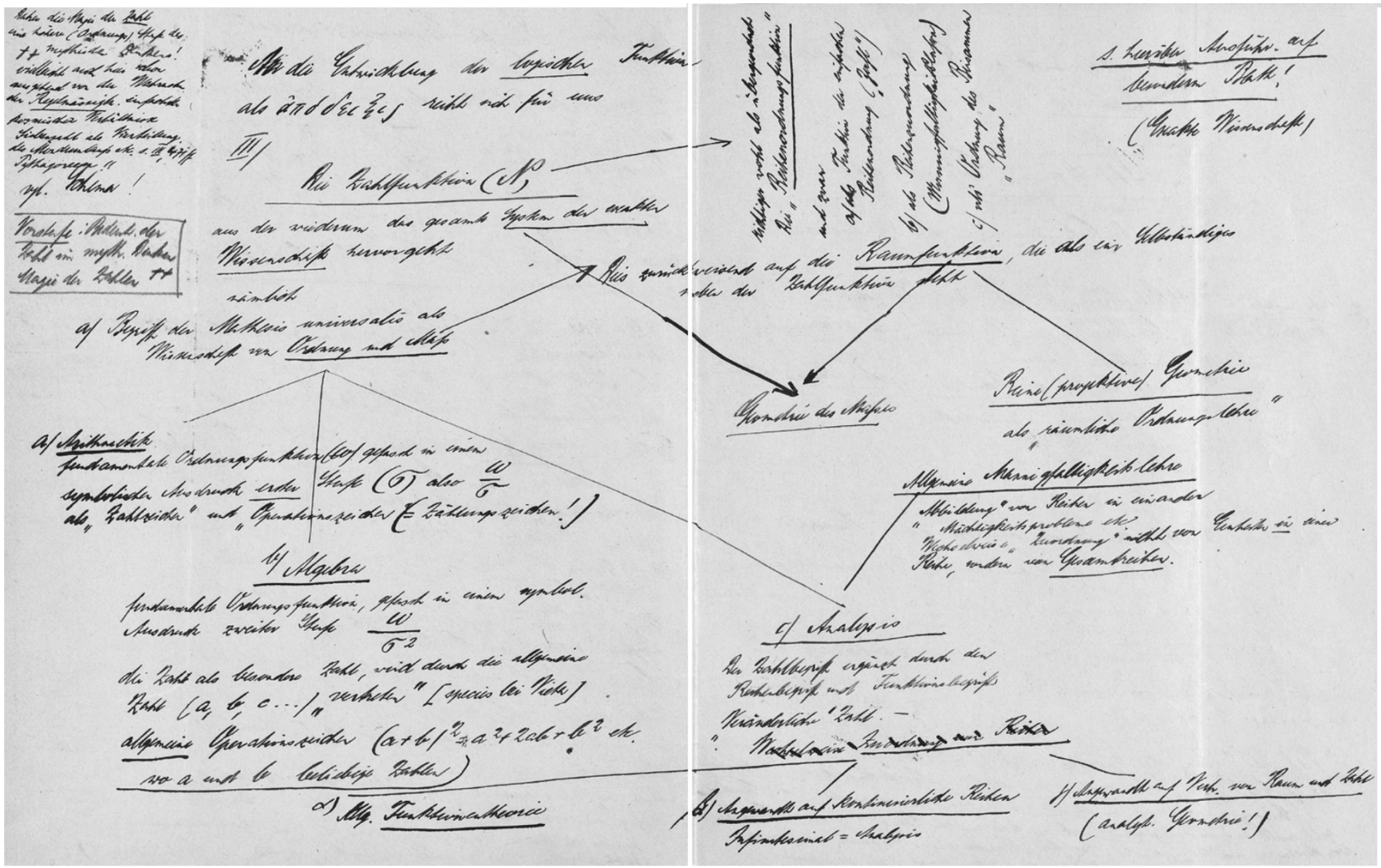
The Energy Principle and Arithmetization of Physics

- **reception** of *Substanzbegriff und Funktionsbegriff* was slow, success via *Das Erkenntnisproblem*
- all attempts to appoint Cassirer as Cohen's **successor** in Marburg after his retirement in 1912 failed. Cassirer remained a *Privatdozent* in Berlin for nearly two decades.
- during those years, Cassirer moved beyond epistemological questions (Cassirer 1914) toward the **philosophy of culture**, publishing *Freiheit und Form* in 1916.
- it was around that time that Cassirer began working on a program of '**Philosophy of the Symbolic**', as evidenced by a 32-page *Disposition* dated June 13, 1917

> "the number function [*Zahlfunktion*] (N), from which the entire system of the exact sciences in turn arises"

Cassirer 1917, 18/19

The Energy Principle and Arithmetization of Physics



Part III

Relativity Theory and the Arithmetization of Geometry

- Cassirer began to pursue the philosophy of the symbolic systematically $\Rightarrow$ manuscript on the **philosophy of language** (Cassirer 1919) shows that, in the ensuing years,
- The project was temporarily put on hold $\Rightarrow$ the **confirmation of general relativity** in November 1919.

Relativity Theory and the Arithmetization of Geometry

Philosophische Probleme der Relativitätstheorie*
von Ernst Cassirer

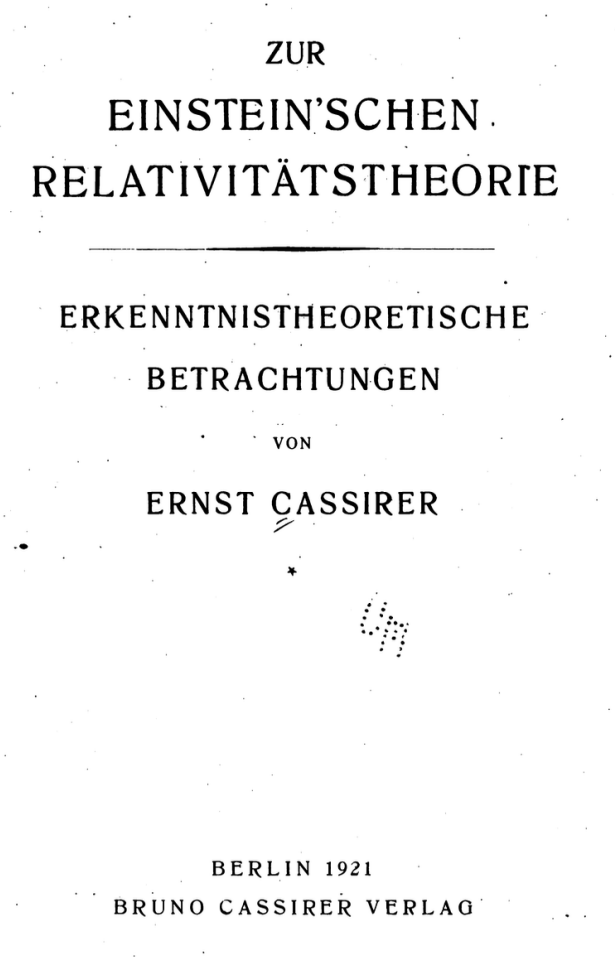
Die philosophische Betrachtung einer physikalischen Theorie kann nicht darauf ausgehen, einen eigenen und selbständigen Maßstab für die Beurteilung ihres Inhalts aufzustellen, der den Maßstäben, über welche die Einzelwissenschaft selbst verfügt, gleichberechtigt zur Seite treten könnte. Denn der Inhalt einer physikalischen Theorie untersteht nur einer einzigen Regel, die sich rein aus der Methodik der Physik als solcher ergibt. Neben dieser Norm bleibt für eine andere rein „spekulative" Betrachtungsweise kein Raum. Mathematische und mathematisch-physikalische Lehrsätze lassen sich von der ihnen gemäßen Darstellungsweise nicht ablösen, ohne damit auch den besten Teil ihres Gehalts zu verlieren. Was ein solcher Lehrsatz bedeutet, das ergibt sich erst, wenn man zugleich mit ihm den gesamten immanenten Begründungszusammenhang erfaßt, in welchem er steht und der ihm erst seinen geistigen Sinn und sein geistiges Leben verleiht. Auch die Relativitätstheorie läßt sich daher von dem Grund und Boden, aus dem sie erwachsen ist, von dem Boden der Experimente und der mathematischen Theorie, nicht abtrennen. Eine rasche Erweiterung dieser Theorie über ihr ursprüngliches Gebiet hinaus, — eine Wendung ihrer Ergebnisse ins schlechthin Allgemeine, ins Spekulativ-Metaphysische mag reizvoll und lockend erscheinen — aber sie käme einer Vernichtung ihres eigentlichen, ihres prinzipiellen Sinns gleich. Auch hier gilt das bekannte Wort eines antiken Mathematikers, daß es keinen „Königsweg zur Mathematik" gibt. Der Weg zum Verständnis der Ergebnisse der

* Der folgende Aufsatz versucht, auf eine Aufforderung der Redaktion der „Neuen Rundschau" hin, den rein philosophischen Kern der Relativitätstheorie herauszuschälen. Der mathematisch-physikalische Inhalt der Theorie konnte hier nur ganz kurz berührt werden; eine eingehende Darlegung der mathematisch-physikalischen Voraussetzungen und eine nähere Begründung der hier nur in knappstem Umriß entwickelten erkenntnistheoretischen Grundauffassung ist in meiner soeben erschienenen Schrift: Zur Einsteinschen Relativitätstheorie. Erkenntnistheoretische Betrachtungen (Berlin, Bruno Cassirer) enthalten.

85 1337

- the ultimate goal of science is to construct "***a pure world of numbers*** [. . .] out of the world of immediate intuition" (Cassirer 1920, 1354).
- "In this relation to the **fundamental motif of number** [*Zahl*], space and time also appear unified in a new way" (Cassirer 1920, 1354).

Relativity Theory and the Arithmetization of Geometry



ZUR

EINSTEIN'SCHEN

RELATIVITÄTSTHEORIE

ERKENNTNISTHEORETISCHE

BETRACHTUNGEN

VON

ERNST CASSIRER

BERLIN 1921

BRUNO CASSIRER VERLAG

- **physics of models** (mechanism)
- **physics of principles** (energetics)

relativity continues
in the latter tradition

Relativity Theory and the Arithmetization of Geometry

> The ideal with which scientific physics began in Pythagoras and the Pythagoreans here finds its completion: all qualities, including those of pure space and pure time, have been transformed into pure numerical values [...] The theory of relativity, however, pushes the dissolution of the form, of experiential quality significantly further, and it does not rest until it has succeeded in dissolving even the fundamental distinction between space and time into one of mere number. [...][Spacetime forms] a four-dimensional manifold: that is, each point within it (each individual 'event') is fully determined by specifying four numerical values x_1, x_2, x_3, x_4. These values have no intuitive or directly physical meaning; rather, as Einstein explicitly states, they serve only 'to assign numbers to the points of the continuum in a specific but arbitrary manner'.
>
> And in this arbitrary numbering, the individual numbers used, x_1, x_2, x_3, x_4, exhibit no intrinsic differences among themselves. *They are nothing but numbers*

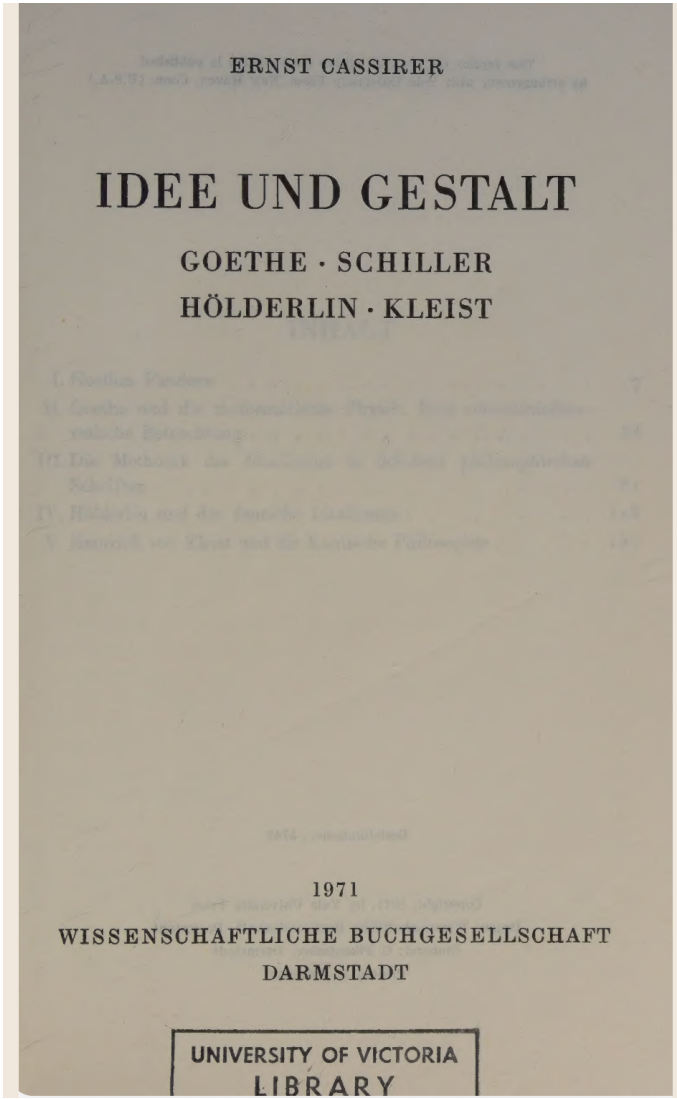
(Cassirer 1921b, 55; tr. 1923, 235)

Relativity Theory and the Arithmetization of Geometry

- a progressive **deanthropomorphization** (Planck), elimination of sensuous qualities (Cassirer 1921b, 116; tr. 1923, 445).
- relativity the result of **degeometrization**, elimination of the intuitive notions of space and time

> “‘Space’ and ‘time’ likewise belong to that content which, like the content of immediate sensation, disappears in the final logical–mathematical form of physics. What remains are only statements about numerical values [*Zahlwerte*] and certain functional correspondences, certain ‘coincidences’ among these numerical values.”
>
> *(Cassirer 1920–1921, 44)*

Relativity Theory and the Arithmetization of Geometry

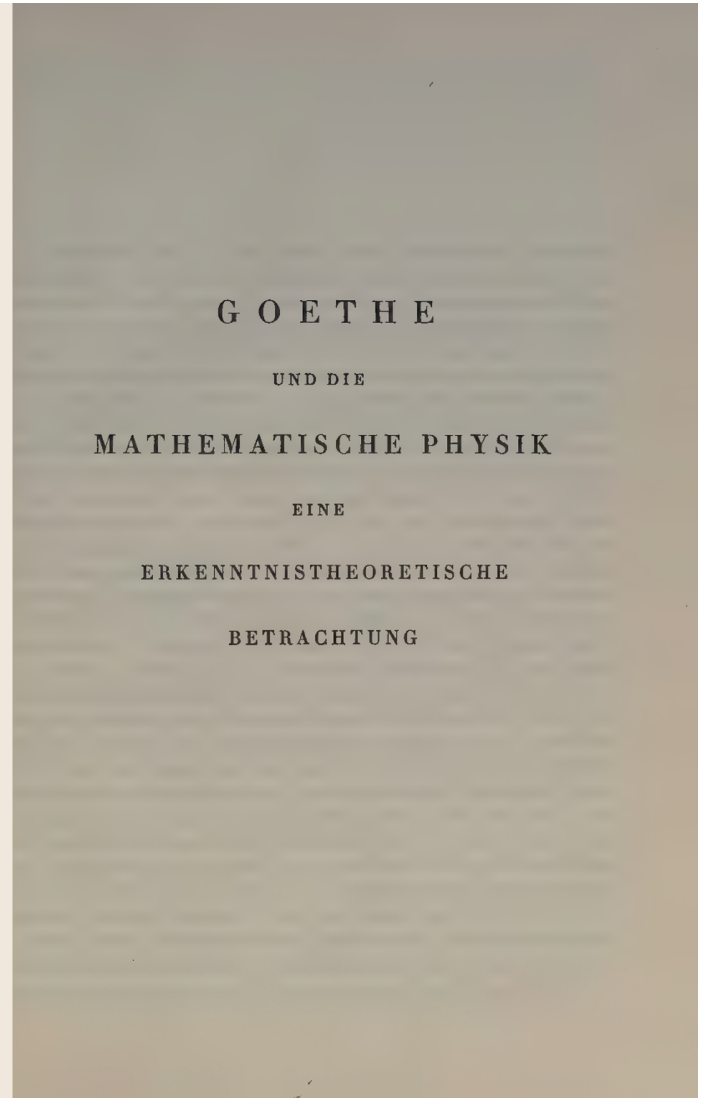


ERNST CASSIRER

IDEE UND GESTALT

GOETHE · SCHILLER
HÖLDERLIN · KLEIST

1971
WISSENSCHAFTLICHE BUCHGESELLSCHAFT
DARMSTADT



GOETHE

UND DIE

MATHEMATISCHE PHYSIK

EINE

ERKENNTNISTHEORETISCHE

BETRACHTUNG

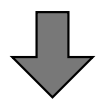
> [A]s nineteenth- and twentieth-century physics advanced, it sought increasingly to discard hypotheses and images [*Bilder*] in order to ground its entire structure all the more firmly and exclusively in pure numerical determinations [*Zahlenbestimmungen*] That such determinations constitute the true framework of 'nature' was the fundamental idea that guided Robert Mayer in developing the general theory of energetics. Yet the reduction to numerical values leaves nothing, in a qualitative sense, of the empirical intuition with which physical inquiry begins. It strives to transform the initial qualitative features into pure positional values [*Stellenwerte*], characterized by nothing other than their relations to other elements in the sequence [*Reihe*] All individuality of intuition is absorbed into such relations; [. . .] The development of the theory of relativity shows how the process of detaching from the particularity of intuitive elements extends beyond sensation itself to the 'forms of pure intuition,' to space and time.

(Cassirer 1921a)

Relativity Theory and the Arithmetization of Geometry

Reihenprinzip (sequence-principle):

- **mathematical physics:** it derives "its sequence principle from the domain of number, the abstract fundamental form of sequence as such" (Cassirer 1921a, 51).
  - *Zahlenreihe*
- **Goethean physics**: it derives his sequence principle from the "phenomenon of life", "without this form of the series requiring the detour through the analytical conceptual tool of number" (Cassirer 1921a, 51)
  - *Lebensreihe*



'selection principle' (*Auswahlprinzip*)

Arithmetization and the Symbolic Nature of Scientific Knowledge

- all concept-formation is characterized by a specific form of '**sequencing**' (*Reihung*) (Cassirer 1922, 7)
- each individual element is **symbol** carrier of the universal, insofar as it is regarded as a member of a lawfully generated sequence of possible elements

number-symbols paradigmatic $\Rightarrow$ de-
fined only by their position in a sequence

PHILOSOPHIE DER SYMBOLISCHEN FORMEN

VON

ERNST CASSIRER

ERSTER TEIL:
DIE SPRACHE

1923

BRUNO CASSIRER VERLAG BERLIN

PHILOSOPHIE DER SYMBOLISCHEN FORMEN

VON

ERNST CASSIRER

ZWEITER TEIL:
DAS MYTHISCHE DENKEN

1925

BRUNO CASSIRER VERLAG BERLIN

PHILOSOPHIE DER SYMBOLISCHEN FORMEN

VON

ERNST CASSIRER

DRITTER TEIL:
PHÄNOMENOLOGIE DER ERKENNTNIS

1929

BRUNO CASSIRER VERLAG BERLIN

> The development of modern theoretical knowledge of nature has carried this ideal of knowledge toward its fulfillment by allowing not only the specific character of sensory perception but also the specific nature of the pure forms of intuition—the nature of space and time—to **dissolve into pure number**. [. . .] Individual numerical entities display no differences among themselves other than those that arise from their **position** within the total system. Accordingly, certain numbers can also be 'defined', that is, **constructively generated**, which—although no specifiable sensory or intuitional substrate corresponds to them directly—are uniquely characterized by these relations [. . .][Dedekind]. Pure mathematical thought is in principle able to grasp any 'individual' number, any numerical entity, only in this form.

(Cassirer 1925, 175 sq.)

Prof. Dr. Ernst Cassirer:

Hochansehnliche Festversammlung! Werte Kollegen! Liebe Kommilitonen!

Wenn nach einem alten und schönen Brauch die erste öffentliche Amtshandlung des neuen Rektors darin zu bestehen hat, daß er vor der Gesamtheit der Universität eine Art von Rechenschaft ablegt über den Stand seiner Wissenschaft und über die Richtung seiner eigenen Forschung, so wird dieser Brauch noch zu etwas anderem und zu etwas mehr, als einer bloß traditionellen Forderung, — er wird zu einer sachlichen Notwendigkeit und zu einer inneren Verpflichtung, sobald diese Forschung sich im Kreise der Philosophie, ihres Systems und ihrer Geschichte, bewegt. Denn die Forderung der Rechenschaftsablegung ist der Philosophie nicht von außen her auferlegt, sondern sie ist der reine Ausdruck ihres eigensten Wesens und ihres Grundproblems. Platon sieht den Kern des philosophischen, des eigentlich-dialektischen Denkens in diesem Akt des ständigen und stetigen Rechenschaft-Ablegens, des Λόγον διδόναι. Und diese Selbstbesinnung und Selbstrechtfertigung erstreckt sich nicht nur auf ihre Resultate, auf den Bestand bestimmter Lehrsätze, sondern sie richtet sich in erster Linie auf das Ganze ihrer Fragestellung und auf das Ganze ihrer Methode. Die Philosophie „ist" nur dadurch, daß sie auf jeder Stufe ihrer Entwicklung immer wieder von neuem nach sich selbst, nach ihrem Rechts- und Wahrheitsgrund, nach ihrer eigenen inneren Möglichkeit fragt. Eine solche Frage ist es denn auch, die ich mir in der heutigen Feierstunde stellen möchte. „Formen und Formwandlungen des philosophischen Wahrheitsbegriffs" — so lautet das Thema, über das ich heute zu Ihnen sprechen möchte. Aber ich muß freilich darauf gefaßt sein, daß schon mit dieser ersten Formulierung des Themas der Zweifel gegen dasselbe bei Ihnen einsetzen wird. Denn — so dürfen und so werden Sie mit Recht fragen — gibt es eine Wandlung und Umgestaltung, gibt es eine Metamorphose, die sich nicht

Arithmetization and the Symbolic Nature of Scientific Knowledge

emancipation from geometrical intuition

- **history of mathematics**
  - “[Dedekind] strives to construct the domain of numbers and the continuum by purely logical means [. . .] showing that the system of real numbers can be obtained without any contribution from the ‘pure forms’ of space and time” (Cassirer 1929, 35)
- **history of physics**
  - “The theory of relativity reveals that, in order to achieve a genuine unity of the world-picture, one must disregard not only the particularity of sensory impressions but also that of our spatial–temporal intuition” (Cassirer 1929, 35)

Part V

The Arithmetization of Mathematics and the Arithmetization of Physics: The Exile years

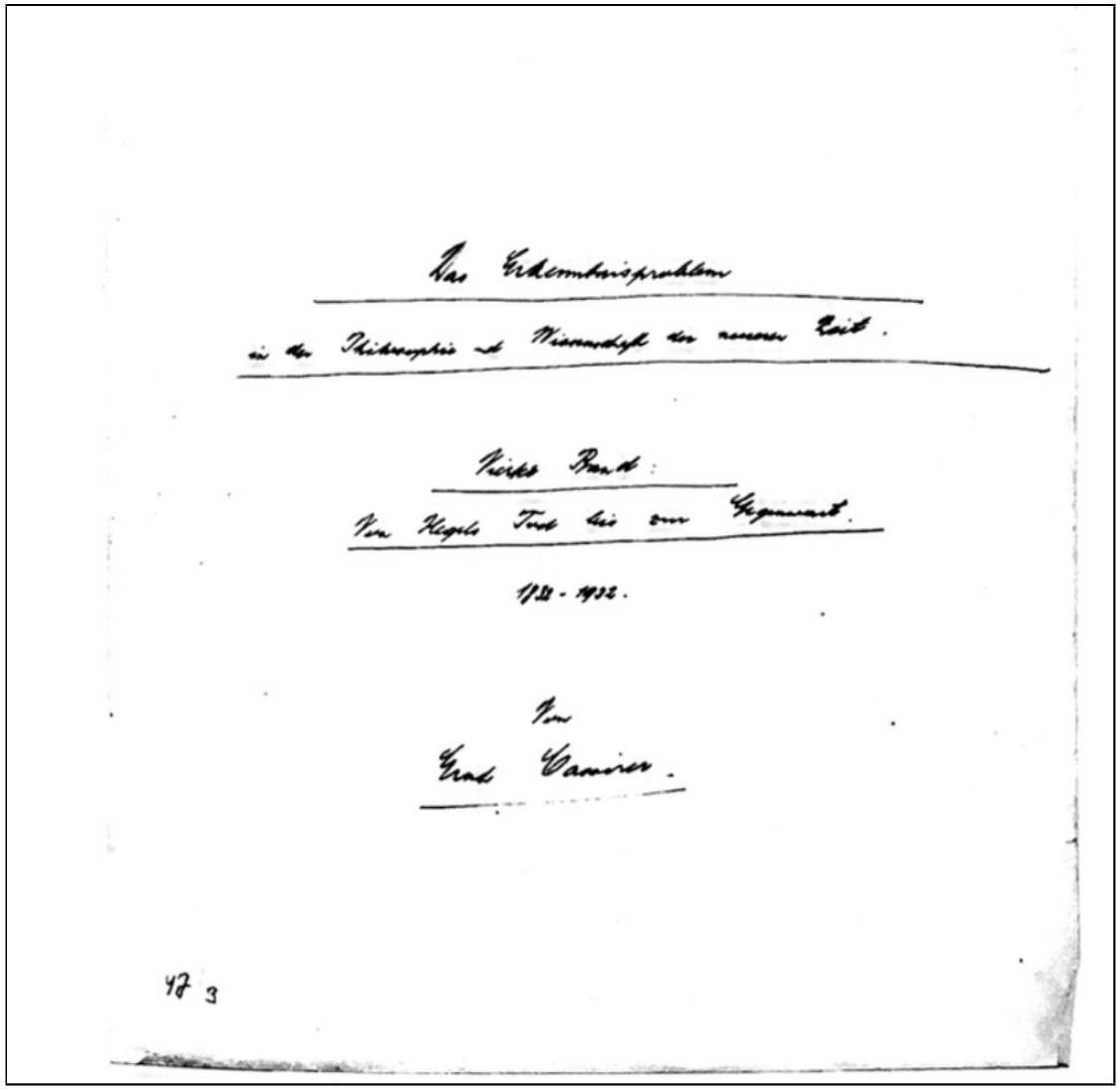
The Arithmetization of Mathematics and the Arithmetization of Physics: The Exile years

- **classical mechanics:** the use of intuitive models becomes progressively irrelevant
- **quantum mechanics:** the use of intuitive models becomes impossible.

> The 'crisis of causality' [*Krise der Kausalität*] brought about by quantum mechanics certainly exists, and it is serious enough. But it is not a crisis of the pure concept of causality; rather, it is a 'crisis of intuition.' [*Krise der Anschauung*] [. . .] From a purely epistemological point of view, this insight is not surprising, since it had already been prepared—before the emergence of modern physics—by developments in nineteenth-century mathematics. Even there, the 'crisis of intuition' had already become manifest.

(Cassirer 1936, 204)

The Arithmetization of Mathematics and the Arithmetization of Physics: The Exile years



Das Erkenntnisproblem

in der Philosophie und Wissenschaft der neueren Zeit.

Vierter Band:

Von Kants Tod bis zur Gegenwart.

1912 - 1932.

Von

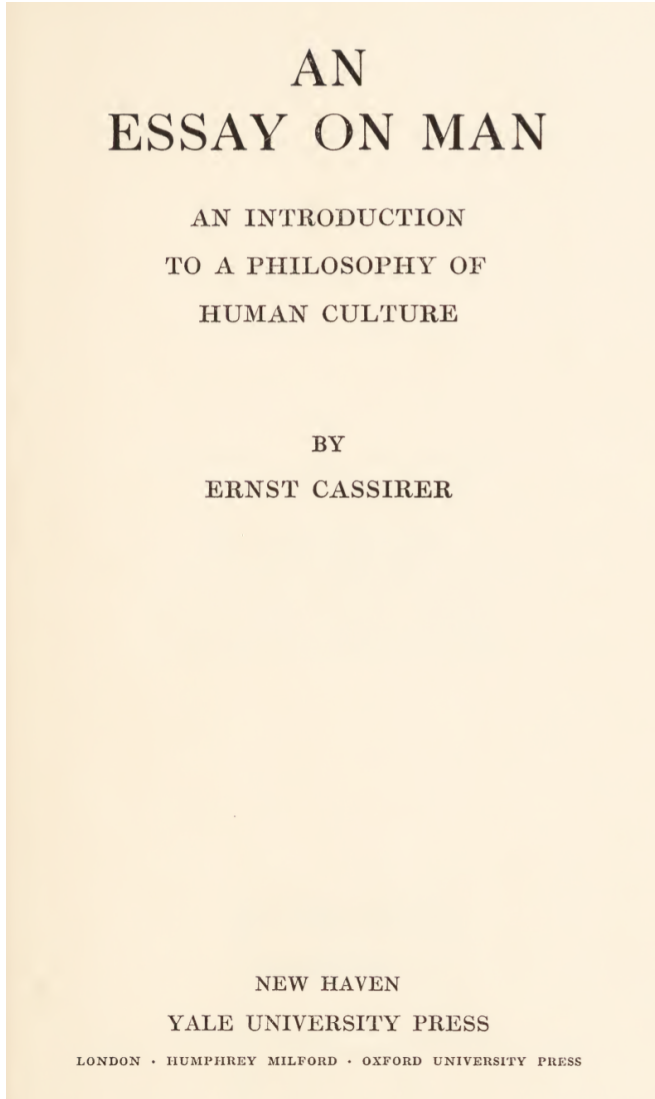
Ernst Cassirer.

47 3

The Arithmetization of Mathematics and the Arithmetization of Physics: The Exile years

- Chapter on the History of Mathematics: 'pathological' cases (Weierstrass's continuous nowhere-differentiable curve, Peano's space-filling curve, etc..) that defy geometrical intuition but are nonetheless validated by arithmetical proof:
  - "Felix Klein treated it as typical of modern mathematics, in that he recognized clearly and carried through completely this demand for the arithmetizing of mathematics" (Cassirer 1940, 66f.; tr. 59).
- Chapter on the History of Physics: Galileo book of nature is written in the language of geometry, Descartes' entirely reducible to geometry. However, starting from the nineteenth century, the tide turned:
  - "Then, however, an intellectual movement set in that may be compared in a way with the arithmetization of mathematics, which occurred at the same time" (Cassirer 1940, 113; tr. 98).

The Arithmetization of Mathematics and the Arithmetization of Physics: The Exile years



AN
ESSAY ON MAN

AN INTRODUCTION
TO A PHILOSOPHY OF
HUMAN CULTURE

BY
ERNST CASSIRER

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> Obviously, this conception is not called into question by any result of modern physics. The progress of quantum mechanics has shown us that our mathematical language is much richer [. . .] than was realized in the systems of classical physics [. . .] But the general form of number is preserved in all these subsequent schemes. In a historical survey of the development of mathematical thought during the nineteenth century, Felix Klein declared that one of the most characteristic features of this development was the progressive 'arithmetization' of mathematics. The same process of arithmetization can also be traced in the history of modern physics. From Hamilton's quaternions to the various systems of quantum mechanics, we find increasingly complex systems of algebraic symbolism. The scientist acts upon the principle that even in the most complicated cases he will eventually succeed in finding an adequate symbolism.
>
> *(Cassirer 1944, 219)*

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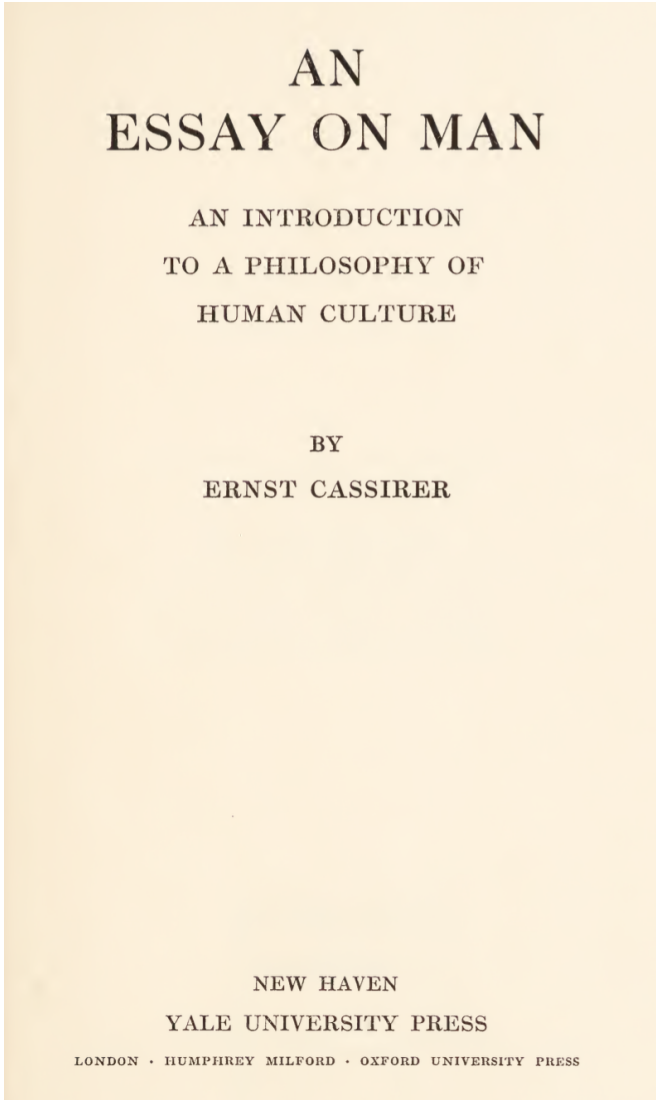

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quantum theory

- **metaphysical determinism** $\Rightarrow$ methodological determinism
- methodological determinism $\Rightarrow$ **numerical determinism**

it must always be possible
to reduce phenomena to
“exact numerical rules”

The Arithmetization of Mathematics and the Arithmetization of Physics: The Exile years

- scientist does not give us a logical or empirical **proof** of this fundamental principle: “The only proof that he gives us is his work”.
- the scientist knows that there are still very large fields of phenomena which it has not yet been found possible to reduce to exact numerical rules. “Nevertheless, he remains faithful to this general Pythagorean **creed**”

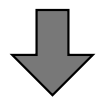
Vertraue und Handle! (Helmholtz)

Part VI

Conclusion

Conclusion

arithmetization of the natural sciences $\Longrightarrow$
historical description and regulative ideal



- a neo-Pythagorean conviction of the '**ontological**' preeminence of numbers as the ultimate constituents of reality
- neo-Kantian assumption of their '**epistemological**' preeminence of numbers as a model of scientific concept-formation

Conclusion

‘concept-formation’ (*Begriffsbildung*) → ‘sequence-formation’ (*Reihenbildung*) → ‘number-sequence’ (*Zahlenreihe*)

- **structuralism**: each actual individual number is defined not by intrinsic content but by its relation to all other numbers;
- **constructivism**: this relation is a productive one, namely the successor function, which generates the sequence of all possible numbers.

- common experience classifies actual perceptible colors
- physics produces the sequence of possible wavelength values

Thanks!

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